Strategic Investment Timing Under Profit Complementarities

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1. Introduction

Strategic competition among rival firms, intensive innovation and a high degree of system complementarity are distinctive and related features of modern information-based economic systems. More specifically, as far as system complementarity, two decades of rapid technological changes resulted in the complementary use of many products, i.e. the utility of the basic component parts of a system is greatly reduced if not combined with some complementary product or service. In fact, it is common to high technology industries to see products that are useless unless combined into a system with other products. Hardware is useless without software, DVD players are useless without content, and an operating system is useless without applications. Very often, moreover, there are externalities derivig from the first firm's investment. For firms operating in these industries, the adoption of new technologies is a crucial component of the investment policies. A number of questions arise: Which kind of adoption pattern will be generated through competition? How this pattern will depend on both system complementarities and externalities generated by early adoption?

Aim of this paper is to address the above questions by analysing and quantifying the effect of the irreversible adoption of a new technology whose returns are uncertain, when firms have an advantage to investing when others also invest and there are externalities from adoption. We derive the optimal adoption timing by applying the real option approach of modern investment theory.

The analysis of strategic investment decisions within the real option approach in a game theoretical setting has been the subject of intense re-

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search interest. Recent surveys of real options and strategic competition may be found in Boyer *et al.* (2004) and Marseguerra - Cortelezzi (2006). More specifically, our work is closely related to Fudenberg - Tirole (1985), Smets (1991), Huisman - Kort (1999, 2004), Kong - Kwok (2007) and Mason - Weeds (2010). Fudenberg - Tirole (1985) present the first strategic investment model that analyses the effect of pre-emption in games of timing in the adoption of a new technology by two competing firms. Though their model assumes zero market uncertainty, their technique of relating the threat of pre-emption to rent equalization forms the cornerstone of the subsequent analysis of strategic investment model. Rent equalization is the fundamental principle according to which the benefits of the leader and of the follower are equal, i.e. the leader invests as soon as demand reaches the pre-emption point. In a subsequent work, Smets (1991) introduces uncertainty and develops a continuous time stochastic duopoly timing game to analyse strategic interactions among firms' investment decisions. Huisman - Kort (1999) extend Smets (1991) allowing for both firms operating in the same output market before investment. Their main result is that in the pre-emption equilibrium situations occur where it is optimal for one firm to invest but at the same time investment is not beneficial if both firms decide to do so. However, being the firms identical, the possibility arises for both firms simultaneously investing and receiving a low pay-off. They find that such a coordination failure can occur with positive probability when the leader's payoff is strictly greater than the follower's pay-off. Huisman - Kort (2004) focus on the strategic effect of the adoption of a new technology. A firm that invests today bears the risk of a much better technology becoming available at an unknown future time. This implies an incentive to delay the investment, thus increasing the option value of waiting to invest in the current technology. On the other hand, the fear of being pre-empted by a competitor may induce the firm to invest quickly disregarding future technological progress. These considerations could turn a pre-emption game into a war of attrition, when the first mover invests in the current technology while the second mover waits for the new technology to arrive and invests then in it. Kong - Kwok (2007) examine strategic investment games between two firms that compete for optimal entry in a project with uncertain revenue flows. They introduce asymmetry on both the sunk cost of investment and revenue flows and provide a complete characterization of pre-emptive, dominant and simultaneous equilibria. In particular, they show that under negative externalities a firm may reduce the loss of real options value by selecting appropriate pre-emptive entry, while under positive externalities firms do not compete to lead. Mason - Weeds (2010) develop an irreversibile investment two-player dynamic model with strategic interactions and externalities between investing agents. As is well known (see Marseguerra - Cortelezzi - Dominioni, 2006), under the possibility of pre-emption a firm's entry point is influenced by two opposite forces: the desire to wait for optimal entry and the urge to preempt. Mason - Weeds show that the leader's investment trigger is bounded above as uncertainty increases and under certain parameter values greater uncertainty may even force the leader to invest earlier.

Our model departs from the reviewed literature in several directions. First, we consider two firms already active on the output market, while most of the literature is concentrated on the new market model. Second, we consider a two-person game of irreversible investments under uncertainty. Like Huissman - Kort (1999) we consider technological uncertainty but we add system complementarities. Finally, unlike Mason - Weeds (2010), we concentrate our analysis on the direct effect of complementary products, while they study the indirect network effect.

The rest of the paper is organised as follows. Section 2 formalises the assumptions of the model. Section 3 analyses the interaction between two investing firms when the order of actions is exogenously given. In Section 4 the pre-emption equilibrium is examined whilst Section 5 derives the cooperative adoption case as a benchmark for the subsequent efficiency analysis developed in Section 6. Finally, Section 7 concludes.

2. The Model

Let us consider two symmetric and risk neutral firms engaged in dynamic competition over an infinite horizon. Both firms make use of an established technology and have access to a new technology which will improve the equilibrium profits of the adopting firm. They must decide whether and when to switch from the existing technology to the new one, i.e. firms hold the option to invest in the new technology. Finally, we consider firms already active on the market.

One of the main features of our model is represented by the assumption relative to complementarities in the product market. The notion of profit complementarity is a cumulative measure of several effects. Demand-side complementary products and economies of scope will generate profit complementarity, whereas demand-side substitution and diseconomies of scope will generate negative profit complementarity. Even if the precise definition of profit complementarity may vary by context, for technology adoption situations like those examined in the present paper it suffices to define the investment strategies made by the two firms as profit positive (negative) complements if the adoption of the technology by one firm increases (decreases) the profitability of the other.

Let $\pi(x, y)$, $x, y \in \{1, 2\}$, be the equilibrium payoff for a given firm where the first argument denotes the technology adopted, either the existing technology (1) or the new available technology (2) for that firm, and the second argument pertains to its competitor. To adopt the innovation, firms incur a sunk cost I, independent of the adoption timing. We assume I to be equal for both firms. These payoffs are quite general and apply both to product and process innovation. In the last case, they can be interpreted as the outcome of an irreversible investment in R&D to lower the production costs. As far as the per period equilibrium profits, the following relations hold:

(1) $$\pi(1, 1) < \pi\ (2, 1)$$

(2) $$\pi(1, 2) < \pi\ (2, 2)$$

(3) $$\pi(2, 1) < \pi\ (2, 2)$$

Assumption 1 formalises the notion that a successful implementation of the new technology yields higher profits. In other words, each firm can make more profits when it produces with the more efficient technology. Assumptions 2 and 3 state that once one of the two firms adopted the efficient technology, joint adoption is preferable for both firms, that is there are profit complementarities in the product market. We leave unspecified the relation between $\pi(1, 2)$ and $\pi(1, 1)$ to allow for both positive ($\pi(1, 1) < \pi(1, 2)$) and negative ($\pi(1, 1) > \pi(1, 2)$) externalities[1]. Thus a firm, when its rival starts using the more efficient technology, may make either more profits (positive externality) or less profits (negative externality).

Uncertainty comes from demand side and it is denoted with Y_t as a multiplicative shock on per period equilibrium profit. We assume that Y_t follows a geometric Brownian motion:

(4) $$dY_t = Y_t \mu d_t + Y_t \sigma dw$$

where $\mu < r$, is the drift parameter[2], r is the risk free discount rate, $\sigma > 0$ is the volatility parameter, and dw is the increment of a Wiener process. Thus dw is distributed according to a normal distribution with mean zero and variance dt. There is complete information about all the relevant parameters of the model.

The game proceeds as follows. In absence of action taken by either firm, the stochastic process evolves according to (4). If one of the two firms has not invested at any time $\tau < t$, its action set is A_t = {invest, don't invest}. If, on the other hand, it has invested at some $\tau < t$, than A_t = {don't invest}. Thus, each firm faces a control problem in which its only choice is when to

[1] Following Dasgupta (2003), by externalities we mean «the side effects of human activities when they are undertaken without mutual agreement. Externalities are often called "spillovers"». For a recent analysis of externalities within the standard new economy geography model, see Nocco (2007).

[2] The restriction ensures that there is a positive opportunity cost to holding the option to invest, and so the option is not held indefinitely.

choose the action «stop». After taking this action, the firm can make no further move to influence the outcome of the game[3].

As usual for dynamic games, the game is solved backwards using subgame perfection. In particular, we are looking for Markov perfect equilibria (MPE) in pure strategies.

In what follows, two models are studied (as in Marseguerra - Cortelezzi - Dominioni, 2006). In the first, there is no pre-emption effect since one agent is exogenously assigned the role of investing first. By ruling out the possibility of pre-emption, this allows to concentrate on the option value of waiting under complementarities and externalities. In the second model, instead, the order of investment is endogenously determined.

3. Equilibrium with exogenous order of actions

This model is well suited to study cases in which one firm has a clear advantage in the adoption of a new technology, e.g. it may be technically more literate, have a more flexible organization, or be less dependent on an existing technology. For example, Bresnahan (1998) argues that there are strong first-mover advantages in high tech sectors because early firms in the computer software market are able to set standards[4].

3.1. *Sequential Investment*

Let us start by assuming that the preassigned leader and follower invest at different instants. The possibility of simultaneous investment is considered in the next subsection. Denoting with $i = L, F$ respectively the leader and the follower payoffs, in what follows we derive the expected total discounted profits of the leader and of the follower. As usual in dynamic games, the stopping time game is solved backwards, in a dynamic programming fashion.

The Follower's Problem. Let us first value the payoff of being a follower. It has three different components holding over different ranges of Y. The first, $F_0(Y)$, describes the value of investment before the leader has invested; the existing technology yields a profit $Y\pi(1, 1)$ per unit of time and its present discounted value is $\frac{Y\pi(1, 1)}{r-\mu}$. Moreover, the follower holds the option to

[3] See Fudenberg - Tirole (1985) and Dutta - Rustichini (1991), for details.

[4] Moreover, Bessen - Maskin (2009) argue that in certain industries, and in particular in the high-tech sectors of computer software and hardware, innovation is both sequential and complementary.

exchange the existing technology for the new one conditional to the leader having already invested. The option to invest should be valued accordingly. In the second region, $F_1(Y)$, the leader has already adopted the new technology, and the follower has to choose his adoption strategy to maximise his option's value. Thus, the value of the follower can be characterised as a portfolio containing the existing technology, yielding a profit $Y\pi(1, 2)$ per unit of time and a present discounted value of $\frac{Y\pi(1,2)}{r-\mu}$, plus an option to exchange the existing technology with the new one. Finally, in the third region, $F_2(Y)$, paying an irreversible adoption cost I, the follower can adopt the new technology and obtain an instantaneous profit $Y\pi(2, 2)$, with a present discounted value of $\frac{Y\pi(2,2)}{r-\mu}$. The follower's value function, $F(Y)$, is thus given by:

$$F(Y) = \begin{cases} \frac{Y\pi(1,1)}{r-\mu} + B_0 Y^{\beta} & Y < Y_L \\ \frac{Y\pi(1,2)}{r-\mu} + B_1 Y^{\beta} & Y \in [Y_L, Y_F) \\ \frac{Y\pi(2,2)}{r-\mu} - I & Y \geq Y_F \end{cases} \tag{5}$$

where, following Dixit - Pindyck (1994), the value matching and smooth pasting conditions are used to determine the critical level of uncertainty which triggers the new technology adoption, Y_F, and the unknown coefficient B_1, whilst the value of the unknown constant B_0 is found by considering the impact of the leader's investment trigger point, Y_L, on the payoff of the follower (see Appendix 1 for a formal derivation). We have:

$$Y_F = \frac{\beta}{\beta-1} \cdot (r-\mu) \cdot I \cdot \frac{1}{\pi(2,2) - \pi(1,2)} \tag{6}$$

Note that Y_F is inversely related to the gain $\pi(2, 2) - \pi(1, 2)$, that is, to the magnitude of the externality caused by the leader investment. The above result is summarized in the following Proposition.

Proposition 1. *Conditional on the leader having adopted the new technology, the optimal follower strategy is to adopt the new technology as soon as Y_t equals or exceeds the trigger value Y_F as given in eq. (6). The corresponding optimal entry timing of the follower, T_F, is:*

$$T_F = \inf\left\{ t \geq 0 : Y \geq \frac{\beta}{\beta-1} \cdot (r-\mu) \cdot I \cdot \frac{1}{\pi(2,2) - \pi(1,2)} \right\} \tag{7}$$

By simple substitution, the value of being the follower is thus given by the following expression:

$$
(8) \qquad F(Y) = \begin{cases} \dfrac{Y\pi(1,1)}{r-\mu} + \left[\dfrac{Y_L[\pi(1,2)-\pi(1,1)]}{r-\mu}\right]\left(\dfrac{Y}{Y_L}\right)^{\beta} + \\ + \left[\dfrac{Y_F[\pi(2,2)-\pi(1,2)]}{r-\mu} - I\right]\left(\dfrac{Y}{Y_F}\right)^{\beta} & Y < Y_L \\ \dfrac{Y\pi(1,2)}{r-\mu} + \dfrac{Y_F[\pi(2,2)-\pi(1,2)]}{r-\mu} \cdot \dfrac{1}{\beta} \cdot \left(\dfrac{Y}{Y_F}\right)^{\beta} & Y \in [Y_L, Y_F) \\ \dfrac{Y\pi(2,2)}{r-\mu} - I & Y \geq Y_F \end{cases}
$$

The Leader's Problem. Let us now consider the value of the leader, $L(Y)$, conditional on the follower pursuing his optimal strategy, in accordance with the rule derived above. As before, there are three components of the leader's value function holding over different ranges of Y. The first, $L_0(Y)$, describes the value of investment before the leader (and so the follower) has invested. The existing technology yields a profit $Y\pi(1, 1)$ per unit of time with a present discounted value $\dfrac{Y\pi(1,1)}{r-\mu}$ and the option to invest given that at some value Y_F the follower will invest too. The second, $L_1(Y)$, describes the situation after the leader has already adopted the new technology giving up the existing one, but before the follower has invested. After investing at $Y < Y_F$, the leader gains duopoly profit $Y\pi(2, 1)$ per unit of time with a present discounted value of $\dfrac{Y\pi(2,1)}{r-\mu}$ and still has the option to invest. Finally, in the third region, $L_2(Y)$, the follower has adopted the new technology, thereby increasing leader's profits to $Y\pi(2, 2)$ per unit of time, with a present discounted value of $\dfrac{Y\pi(2,2)}{r-\mu}$. If the leader invests when $Y \geq Y_F$, the follower will invest too, so that the leader's expected value of the investment is $F(Y)$. The value of the leader, denoted by $L(Y)$, can be expressed as follows:

$$
(9) \qquad L(Y) = \begin{cases} \dfrac{Y\pi(1,1)}{r-\mu} + B_{L0}Y^{\beta} & Y < Y_L \\ \dfrac{Y\pi(2,1)}{r-\mu} + B_{L1}Y^{\beta} - I & Y \in [Y_L, Y_F) \\ \dfrac{Y\pi(2,2)}{r-\mu} - I & Y \geq Y_F \end{cases}
$$

where B_{L0} and B_{L1} are the coefficients of the option value to invest and

$$
(10) \qquad Y_L = \frac{\beta}{\beta - 1} \cdot (r-\mu) \cdot I \cdot \frac{1}{\pi(2,1) - \pi(1,1)}
$$

See again Appendix 1 for the formal derivation of Y_L, B_{L0} and B_{L1}. The following Proposition summarises the result.

Proposition 2. *The optimal leader strategy is to adopt the new technology the first moment that Y_t equals or exceeds the trigger value Y_L, i.e. the optimal entry time of the leader, T_L, can be written as*:

$$T_L = \inf\left\{ t \geq 0 : Y \geq \frac{\beta}{\beta - 1} \cdot (r - \mu) \cdot I \cdot \frac{1}{\pi(2,1) - \pi(1,1)} \right\} \tag{11}$$

By simple substitution we are able to write the leader's value function:

$$L(Y) = \begin{cases} \dfrac{Y\pi(1,1)}{r-\mu} + \left[\dfrac{Y_L[\pi(2,1) - \pi(1,1)]}{r-\mu} - I\right]\left(\dfrac{Y}{Y_L}\right)^{\beta} \\ + \left[\dfrac{Y_F[\pi(2,2) - \pi(2,1)]}{r-\mu}\right]\left(\dfrac{Y}{Y_F}\right)^{\beta} & Y < Y_L \\ \dfrac{Y\pi(2,1)}{r-\mu} + \dfrac{Y_F[\pi(2,2) - \pi(2,1)]}{r-\mu} \cdot \dfrac{1}{\beta} \cdot \left(\dfrac{Y}{YF}\right)^{\beta} & Y \in [Y_L, Y_F) \\ \dfrac{Y\pi(2,2)}{r-\mu} - I & Y \geq Y_F \end{cases} \tag{12}$$

Note that the leader stopping time is inversely related to the gain of being the leader, i.e. $\pi(2, 1) - \pi(1, 1)$. Furthermore, a simple comparison of eqs. (6) and (10) shows that Y_F does not depend on Y_L and symmetrically Y_L does not depend on Y_F. Finally, $Y_L < Y_F$ if and only if $\pi(2, 2) - \pi(1, 2) < \pi(2, 1) - \pi(1, 1)$, that is when the gain of being a leader is greater than the gain of being a follower. If $\pi(2, 2) - \pi(1, 2) \geq \pi(2, 1) - \pi(1, 1)$, then $Y_L \geq Y_F$ and the investment would occur as a *cascade*, that is the leader would invest at Y_L and the follower would invest immediately afterwards. The results are summarised in the following Proposition:

Proposition 3. *When the gain of being a leader is greater than the gain of being a follower, a sequential adoption pattern arises in equilibrium. Otherwise a cascade investment occurs at Y_L.*

As far as the impact of uncertainty on adoption timing, since $\dfrac{\partial\beta}{\partial\sigma} > 0$ from eqs. (6) and (10) it follows that greater uncertainty induces an higher trigger value[5] for both the leader and the follower. Thus, higher uncertainty tends to delay the adoption of the new technology. The intuition for these results has been stressed by McDonald - Siegel (1986) and Pindyck (1988) among the others. Uncertainty combined with a sunk cost creates an option

[5] If a Marshallian rule were used for the investment decision, the trigger point would be simply the cost, i.e. $\dfrac{(r-\mu)I}{\pi(2,2) - \pi(1,2)}$.

value of delaying investment and it should be incorporated in the present discounted value calculation regarding the investment decision. By investing in the new technology, the firm exercises the option and loses the opportunity of waiting for more favorable circumstances. Thus, as in the standard option theory, the profitability of the new technology has to be higher in order to compensate for the possible loss coming from uncertainty. Finally, as for the magnitude of the externality arising from the rival's adoption of the more efficient technology, it does not affect the leader (neither his value function nor his adoption time) but it does affect the follower, who invests earlier the greater the gain from adoption.

3.2. *Simultaneous Investment*

We now consider the special case where both firms decide on a common adoption time T_S, where $T_S = \inf(t \mid Y_t \geq 0)$. Note that here the only aspect of cooperation is the timing of adoption of the new technology and that firms maintain a non-cooperative behavior in the product market – see Mason - Weeds (2010), for this kind of analysis. The value function of each firm, denoted by $S(Y)$, under a simultaneous adoption rule is

$$S(Y) = \begin{cases} \dfrac{Y\pi(1,1)}{r-\mu} + \dfrac{\beta}{\beta-1} I\left(\dfrac{Y}{Y_S}\right) & Y < Y_S \\ \dfrac{Y\pi(2,2)}{r-\mu} - 2I & Y \geq Y_S \end{cases} \tag{13}$$

where

$$Y_S = \frac{\beta}{\beta-1} \cdot (r-\mu) \cdot I \cdot \frac{1}{\pi(2,1) - \pi(1,1)} \tag{14}$$

(see Appendix 1 for formal derivations). The following Proposition holds.

Proposition 4. *The optimal simultaneous adoption strategy is to invest in the new technology as soon as Y_t equals or exceeds the trigger value Y_S as given in eq. (14). The corresponding optimal entry timing can be written as*:

$$T_S = \inf\left\{ t \geq 0 : Y_S \geq \frac{\beta}{\beta-1}(r-\mu) I \frac{1}{\pi(2,2) - \pi(1,1)} \right\}$$

It is worth noticing that $Y_S < Y_L$ since $\pi(2,1) > \pi(1,1)$ by assumption (1). Thus, under profit complementarity and an exogenous order of actions, cooperation on the investment timing is unequivocally socially beneficial. Moreover $Y_S < Y_F$ when there are positive externalities, i.e. if $\pi(1,2) > \pi(1,1)$, and $Y_S > Y_F$ when there are negative externalities, i.e. $\pi(1,2) < \pi(1,1)$. This is quite reasonable: when adoption is viewed within the competition framework, this simpliy says that a positive (negative) spillover induces firms to invest earlier (later) in the simultaneous equilibium.

3.3. *Sequential versus Simultaneous Investment*

We want now to investigate under what conditions will the two firms decide a common adoption time, i.e. when simultaneous rather than sequential/cascade equilibrium will arise. The following Proposition holds:

Proposition 5. *The necessary and sufficient condition for a simultaneous investment to occur in equilibrium is*

$$\delta \equiv \left(\frac{Y_F}{Y_S}\right)^{\beta} - \left(\frac{Y_F}{Y_L}\right)^{\beta} - \beta\frac{\pi(2,2)-\pi(2,1)}{\pi(1,2)-\pi(1,1)} > 0 \tag{15}$$

Proof. See Appendix 1. ■

Whether simultaneous adoption occurs in equilibrium is determined by whether the leader wishes to adopt before the follower, or at the same time (i.e., by comparison of $S(Y)$ and $L(Y)$). Note that the simultaneous adoption equilibrium, when it exists, Pareto dominates the sequential outcome; this is an immediate consequence of the condition for the existence of the simultaneous adoption equilibrium: $S(Y) > L(Y)$ for $Y \in [0; Y_S]$. Figure 1 allows us to have an immediate perception of the parameters' effect[6].

Note that under a first mover advantage and a weakly positive spillover, the equilibrium pattern is a decreasing function of the variance and it switches from a simultaneous equilibrium (for lower values of σ^2) to a sequential equilibrium (for high values of σ^2).

4. Equilibrium with endogenous order of actions

In contrast with the setting considered in the previous Section, assume now that the leader is endogenously determined (i.e. both firms may *a priori* potentially become leader).

4.1. *Sequential Investment*

Without the ability to precommit to trigger points at the beginning of the game, the leader's stopping time cannot be derived as the solution to a single

[6] Parameter values are as follows: The discount rate r has been set equal to 0.08, the investment cost I has been normalised to 1, and the profits are $\pi(1,1) = 4$, $\pi(2,1) = 7$, $\pi(2,2) = 8$ and $\pi(1,2) = 3$ (negative spillover) and $\pi(1,2) = 5$ (positive spillover).

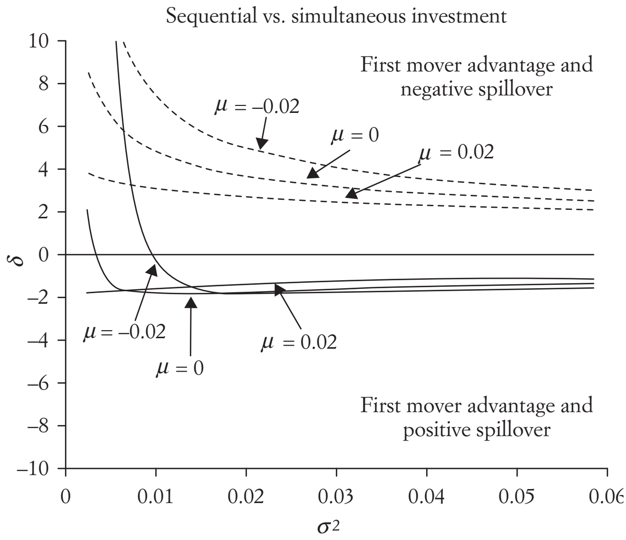

FIG. 1. Equilibrium Analysis (a).

agent optimization problem. Whether a firm becomes a leader and invests, is determined by the firm's incentive to preempt its rival. Thus, the roles of the leader and the follower are determined endogenously. Firms are symmetric before moving, i.e. no firm has a clear advantage from the beginning. As before, let us assume that one firm (the pre-emptor) invests strictly before the other. The follower's value function and trigger point are the same as for the model without pre-emption. The leader's value function is as described in the previous Section. However, the leader can no longer choose its adoption point optimally, as if the roles were preassigned. Instead, the first firm to adopt does so at the point at which it prefers to lead rather than follow[7]. Hence, the adoption point, Y_P, is defined by the indifference between leading and following:

$$L_1(Y_P) = F_1(Y_P)$$

The following Proposition summarizes the result.

Proposition 6. *There exists a unique endogenous cascade equilibrium outcome at* $Y_P \equiv Y_F$.

Proof. See Appendix 2. ■

[7] As in Grenadier (1996) we assume that «if each tries to invest first, one will randomly (i.e. through the toss of a coin) win the race».

4.2. *Simultaneous Investment*

The solution for simultaneous investment in the pre-emption model is the same as in the model without pre-emption: the trigger point is the same, Y_S, and therefore we refer the reader to the previous Section.

4.3. *Sequential versus Simultaneous Investment*

When does simultaneous rather than sequential equilibrium arise? The following Proposition holds:

Proposition 7. *If* $Y_S > (<) Y_P$, *i.e. for* $\pi(1,2) < (>) \pi(1,1)$, *two types of equilibria exist: the endogenous equilibrium described in the previous Proposition and the joint investment equilibrium. Moreover, the joint investment equilibrium Pareto dominates the decentralized equilibrium.*

Proof. See Appendix 2. ■

5. Equilibrium Under Cooperation

This Section analyses the cooperative solution, i.e. when agents' investment trigger points are chosen to maximise the sum of their two value functions. The objective is to provide a benchmark to identify inefficiencies in the next Section.

Let us first examine the case when investment is sequential. The two trigger points, Y_{1L} and Y_{2F}, are chosen to maximise the sum of the leader's and follower's value functions, denoted by $C_{LUF}(Y)$, that is

$$(16) \qquad C_{LUF}(Y) = \begin{cases} \dfrac{2Y\pi(1,1)}{r-\mu} + B_0 Y^\beta + B_1 Y^\beta & Y < Y_{1L} \\ \dfrac{2Y\pi(2,1)}{r-\mu} + B_2 Y^\beta - I + B_3 Y^\beta & Y \in [Y_{1L}, Y_{2F}) \\ \dfrac{2Y\pi(2,2)}{r-\mu} - 2I & Y \geq Y_{2F} \end{cases}$$

where B_k, $k = 0, 1, 2, 3$ are constants to be determined. The cooperative trigger points are determined by the value matching and smooth pasting conditions at both points (see Appendix 3 for a formal derivation) and they are given by

$$(17) \qquad Y_{1L} = \frac{\beta}{\beta - 1} \cdot (r - \mu) \cdot I \cdot \frac{1}{\pi(2,1) + \pi(1,2) - 2\pi(1,1)}$$

(18) $$Y_{2F} = \frac{\beta}{\beta - 1} \cdot (r - \mu) \cdot I \cdot \frac{1}{2\pi(2,2) - \pi(2,1) - \pi(1,2)}$$

The results are summarised in the following Proposition.

Proposition 8. *Conditional on roles exogenously assigned, the optimal leader strategy is to adopt the new technology the first moment that Y_t equals or exceeds the trigger value Y_{1L}, and the optimal follower strategy is to adopt the new technology the first moment that Y_t equals or exceeds the trigger value Y_{2L}. That is, the optimal entry time of the leader, T_{1L}, and of the follower, T_{2L}, can be written, respectively, as*:

(19) $$T_{1L} = \inf\left\{t \geq 0: Y \geq \frac{\beta}{\beta - 1} \cdot (r - \mu) \cdot I \cdot \frac{1}{\pi(2,1) + \pi(1,2) - 2\pi(1,1)}\right\}$$

and

(20) $$T_{2F} = \inf\left\{t \geq 0: Y \geq \frac{\beta}{\beta - 1} \cdot (r - \mu) \cdot I \cdot \frac{1}{2\pi(2,2) - \pi(2,1) - \pi(1,2)}\right\}$$

It is worth noticing that $Y_{1L} \gtreqless Y_{2F}$. Precisely, $Y_{1L} < Y_{2F}$ if $\pi(2, 1) - \pi(1, 1) > \pi(2, 2) - \pi(1, 2)$, and a sequential pattern of adoption of the new technology arises; $Y_{1L} \geq Y_{2F}$ if $\pi(2, 1) - \pi(1, 1) \leq \pi(2, 2) - \pi(1, 2)$ and in this case a cascade investment occurs at Y_{1L}. Let us now consider the cooperative solution with simultaneous investment at the trigger point Y_{3S}. The cooperative value function in this case is

(21) $$C_S(Y) = \begin{cases} \dfrac{2Y\pi(1,1)}{r-\mu} + B_4 Y^{\beta} & Y < Y_{3S} \\ \dfrac{2Y\pi(2,2)}{r-\mu} - 2I & Y \geq Y_{3S} \end{cases}$$

Again, value matching and smooth pasting determine Y_{3S} and simultaneously B_4, given by

(22) $$Y_{3S} = \frac{\beta}{\beta - 1} \cdot (r - \mu) \cdot I \cdot \frac{1}{\pi(2,2) - \pi(1,1)}$$

and

(23) $$B_4 = \frac{2I}{\beta - 1} Y_{3S}^{-\beta}$$

Proposition 9 *In the cooperative solution with simultaneous investment, the optimal strategy is to adopt the new technology the first moment that Y_t*

equals or exceeds the trigger value Y_{3S}, i.e. the optimal entry timing of both firms, T_{3S}, can be written as:

$$(24) \qquad T_{3S} = \inf\left\{t \geq 0 : Y \geq \frac{\beta}{\beta - 1} \cdot (r - \mu) \cdot I \cdot \frac{1}{\pi(2,2) - \pi(1,1)}\right\}$$

Note that in both cases $Y_{3S} < Y_{1L}$ but two different adoption patterns are possible. If $\pi(2, 2) - \pi(1, 2) < \pi(2, 1) - \pi(1, 1)$, then $Y_{3S} < Y_{1L} < Y_{2F}$. If $\pi(2, 2) - \pi(1, 2) > \pi(2, 1) - \pi(1, 1)$, then $Y_{3S} < Y_{2F} < Y_{1L}$. As in the previous Section, the following Proposition spells out the conditions leading to the simultaneous rather than sequential adoption pattern:

Proposition 10. *The necessary and sufficient condition for simultaneous investment to be the cooperative solution is $C_S(Y) > C_{LUF}(Y)$, $\forall\, Y \in [Y_S, Y_L]$, i.e.,*

$$(25) \qquad \delta_c = 2\left(\frac{Y_{2F}}{Y_{3S}}\right)^{\beta} - \left(\frac{Y_{2F}}{Y_{1L}}\right)^{\beta} - (2 - \beta) > 0$$

Proof. See Appendix 3. ■

Figure 2 shows an example. Note that in the cooperative case the simultaneous adoption is always preferred.

6. Inefficiencies

In this Section we analyse the inefficiencies that may arise in the noncooperative equilibria in comparison with the cooperative solution.

Let us first consider the model with the roles exogenously assigned. We identify two inefficiencies in the investment decisions, one for the leader's strategy and the other for the follower's strategy. The leader can invest either too early or too late with respect to the benchmark case of the cooperative adoption. Specifically, $Y_L > Y_{1L}$ if there are positive spillovers, i.e. $\pi(1, 2) > \pi(1, 1)$, and $Y_L < Y_{1L}$ if there are negative spillovers, i.e. $\pi(1, 2) < \pi(1, 1)$. Therefore, conditional on both equilibria involving sequential investment, with respect to the cooperative case in the non-cooperative case the leader will invest later if there are positive spillovers and earlier if there are negative spillovers. On the other hand, the follower always invests later, without considering the effect of the spillovers of the leader strategy. In fact, $Y_F > Y_{2F}$ since $\pi(2, 2) > \pi(2, 1)$ by assumption. Therefore, conditional on both equilibria involving sequential investment, the follower always invests later than in the cooperative case.

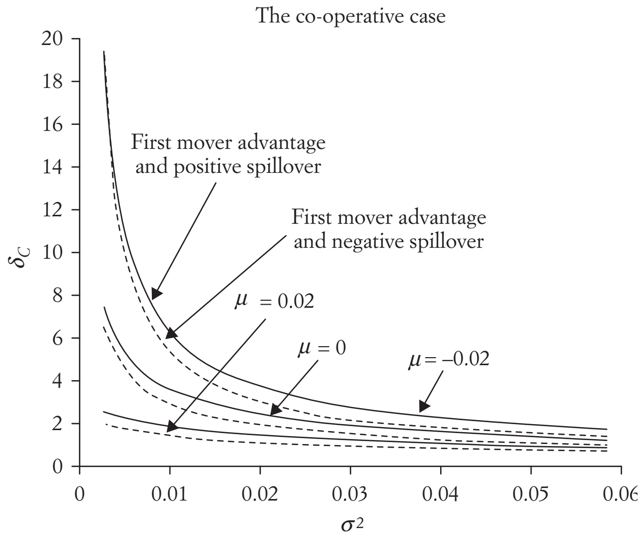

FIG. 2. Equilibrium Analysis (b).

Let us now consider the investment strategies resulting from the model with endogenous roles. Note that $Y_P < Y_{3S}$ if there are negative spillovers, i.e. $\pi(1,2) < \pi(1,1)$, while $Y_P > Y_{3S}$ if there are positive spillovers, i.e. $\pi(1,2) > \pi(1,1)$. Therefore, conditional on both equilibria involving simultaneous adoption, firms will invest either later under negative spillover or earlier under positive spillover. Finally, there are no inefficiencies in the case of simultaneous adoption.

Summing up, cooperation is not necessarily beneficial as far as the leader whilst it is always beneficial as far as the follower. In other words, competition can lead to «too much» waiting, which can be sub-optimal. Thus it is in general unclear whether offering firms the option to cooperate is beneficial, but this analysis provides an argument for a closer look to merger and joint ventures, which may lead to a reduction of investment inefficiencies by internalizing the externality effects.

7. Conclusions

This paper has investigated the combined effect of market structure, profit complementarities and externalities on the adoption of a new and more efficient technology. The analysis is developed within a stochastic dynamic game which allows to consider not only the decision to adopt or not to adopt the technology, jointly or separately, but also the decision concerning when to adopt. We have analysed in particular how investment decisions are affected by the market structure under which firms operate (i.e. non-cooperative vs. cooperative behaviour).

As far as the non-cooperative situation, we focused on two different economic settings. We have first considered the case of a firm with a clear advantage in the adoption of the new technology, so that it can act as pre-designated leader (and the order of actions is therefore exogenously given). In this scenario, if the gain of being the leader is greater than the gain of being the follower, we have shown that a sequential adoption pattern arises in equilibrium. Otherwise a cascade investment occurs. Furthermore, when both firms decide on a common adoption time, and thus they cooperate but only on timing, the (necessarily simultaneous) investment takes place earlier. We derive the precise conditions under which common adoption arises.

The second economic setting we investigated is a situation where the two firms may both potentially invest and adopt the new technology. Neither firm can now be absolutely sure to be the first to adopt and thus now the leader is endogenously determined. In this scenario, we show that there exists a unique cascade equilibrium outcome and, again, we derive the conditions under which common adoption arises. As a benchmark case for analysing market's inefficiencies, we have also derived the model's fully cooperative solutions, i.e. when investment trigger points are chosen so as to maximise the sum of the two firms' value functions. We derive both the sequential (with a pre-assigned leader) and the simultaneous (no pre-assigned leader) adoption patterns and the conditions under which a simultaneous investment equilibrium arises. Comparing the non cooperative with the cooperative solutions, we show that in the non-cooperative case the leader will invest later if there are positive externalities and earlier if there are negative externalities, whilst the follower always invests later. Thus, the results of the comparisons show that in general it is unclear whether offering firms the option to cooperate is beneficial. The analysis here developed however provides an argument for a closer look to merger and joint ventures by precisely spelling out the specific conditions in which these cooperative institutions may lead to substantial investment inefficiencies' reduction.

An interesting extension to our analysis would be to investigate how market structure affects profit complementarities among adoption decisions. This would provide a further link between competition policy and technological diffusion.

8. Appendix 1

This Appendix is devoted to a formal derivation of the follower's and the leader's value functions. First, as far as the follower's value function, note that at each point in time the follower can either invest, and take the termination payoff, or can wait for an infinitesimal time *dt* and postpone the decision. Denoting by $F_{F,1}(Y)$ the option value to invest, the Bellman equation of the problem is

$$F_{F,1}(Y) = Max\left\{V_F(Y) - \frac{Y\pi(1,2)}{r-\mu}, \frac{1}{1+rdt}E\left[F_{F,1}(Y+dY|Y)\right]\right\}$$

where E denotes the expectation, $V_F(Y)$ denotes the firm's equity value, net of the investment cost I. In the continuation region the Bellman equation for the value of the investment opportunity, $F_{F,1}(Y)$, is given by

$$rF_{F,1}dt = E(dF_{F,1})$$

Expanding $dF_{F,1}$ using Ito's lemma and after some simple substitution, the Bellman equation entails the following second-order differential equation

$$\frac{1}{2}\sigma^2 Y^2 F''_{F,1}(Y) + \mu Y F'_{F,1}(Y) - rF_{F,1}(Y) = 0$$

From (4) it can be seen that if Y ever goes to zero it stays there forever. Therefore $F_{F,1}(Y)$ must satisfies the following boundary condition

$$F_{F,1}(0) = 0$$

and the general solution for the above differential equation is

$$F_{F,1}(Y) = B_1 Y^\beta + B_2 Y^\lambda$$

where $\beta > 1$ and $\lambda < 0$ are respectively the positive and the negative root of the fundamental characteristic equation[8] $Q(z) = \frac{1}{2}\sigma^2 z(z-1) + \mu z - r$, and B_1 and B_2 are unknown constant to be determined. Imposing the previous boundary condition the value of the option to invest is

$$F_{F,1}(Y) = B_1 Y^\beta$$

and the option value of waiting is

$$F_1(Y) = \frac{Y\pi(1,2)}{r-\mu} + B_1 Y^\beta$$

The value in the first region is derived applying the same procedure. The value of the option to invest is $F_{F,0} = B_0 Y^\beta$, and the expected value of the

[8] See Dixit - Pindyck (1996, pp. 142-143), for details.

firm if it would never invest is $\frac{Y\pi(1,1)}{r-\mu}$. By summing up these two components gives

$$F_0(Y) = \frac{Y\pi(1,1)}{r-\mu} + B_0 Y^{\beta}$$

that is the option value of waiting in the first region. We next consider the value of the firm in the stopping region, $F_3(Y)$. Since investment is irreversible, the value of the agent in the stopping region is given by the expected value alone with no option value terms, i.e.

$$F_3(Y) = \frac{Y\pi(2,2)}{r-\mu} - I$$

Putting together the three regions gives the follower's value function $F(Y)$ (eq. (5) in the text). Following Dixit and Pindyck (1994) the two components of the follower's value function $F_0(Y)$ and $F_1(Y)$ have to meet smoothly at Y_F with equal first derivatives, and, together with the value matching condition, this implies

$$\begin{cases} \dfrac{Y\pi(1,2)}{r-\mu} + B_1 Y^{\beta} = \dfrac{Y\pi(2,2)}{r-\mu} - I \\ \dfrac{\pi(1,2)}{r-\mu} + B_1 \beta Y^{\beta-1} = \dfrac{\pi(2,2)}{r-\mu} \end{cases}$$

Solving the above system, we can compute the value of the optimal trigger point Y_F (eq. (6) in the text) and the unknown constant B_1, i.e.

$$B_1 = \frac{\pi(2,2) - \pi(1,2)}{r-\mu} \cdot \frac{1}{\beta} \cdot Y_F^{1-\beta}$$

When Y_L is first reached, the leader invests and the follower payoff is altered either positively or negatively. Since the value functions are forward-looking, $F_0(Y)$ anticipates the effect of the leader's action and must therefore meet $F_1(Y)$ at Y_L. Hence, a value matching condition holds at this point; however there is no optimality on the part of the follower, and so no corresponding smooth pasting condition. This implies that

$$B_0 = \frac{Y_L[\pi(1,2) - \pi(1,1)]}{r-\mu} Y_L^{-\beta} + \left[\frac{Y_F[\pi(2,2) - \pi(1,2)]}{r-\mu} - I\right] Y_F^{-\beta}$$

By simple substitution we get eq. (8) in the text. As far as the value of being the leader, denoted by $L(Y)$, it can be expressed as follows:

$$L(Y) = \begin{cases} L_0(Y) = E\left[\int_0^{T_L} e^{-rt} Y\pi(1,1)dt\right] + & \\ E\left[\int_{T_L}^{T_F} e^{-rt} Y\pi(2,1)dt\right] + E\left[\int_{T_F}^{+\infty} e^{-rt} Y\pi(2,2)\,dt\right] & Y < Y_L \\ L_1(Y) = E\left[\int_t^{T_F} e^{-rt} Y\pi(2,1)dt\right] + E\left[\int_{T_F}^{+\infty} e^{-rt} Y\pi(2,2)\,dt\right] & Y \in [Y_L, Y_F) \\ L_2(Y) = V_F(Y) & Y \geq Y_F \end{cases}$$

Let us first derive the second component of $L_1(Y)$. We define the first term of equation $L_1(Y)$ as

$$f(Y) = E\left[\int_t^{T_F} e^{-r(t-\tau)}[Y\pi(2,1)]d\tau\right]$$

Applying the Bellman principle, we get

$$f(Y) = \pi(2,1)dt + \frac{1}{1+rdt}E[Y + dY|Y]$$

Expanding the right hand side using Ito's Lemma and rearranging, we obtain the following differential equation

$$\frac{1}{2}\sigma^2 Y^2 f''(Y) + \mu Y f'(Y) + f(Y)r + \pi(2,1) = 0$$

whose general solution is $f(Y) = A_1 Y^\beta + A_2 Y^\lambda + \frac{\pi(2,1)}{r-\mu}$. Note that if Y approaches Y_F, than T_F goes to zero, and if Y approaches zero, than it will remain at zero for T_F becoming infinitely large. Therefore, the following boundary conditions apply:

$$f(Y_F) = 0$$

$$f(0) = \frac{\pi(2,1)}{r-\mu}$$

It follows that $A_2 = 0$ and $A_1 Y_F^\beta + \frac{\pi(2,1)}{r-\mu} = 0$. By simple substitution, we obtain

$$f(Y) = -\frac{\pi(2,1)}{r-\mu}\left(\frac{Y}{Y_F}\right)^\beta + \frac{Y\pi(2,1)}{r-\mu}$$

The same procedure applies to determine the second term of $L_1(Y)$, $L_0(Y)$ and $L_2(Y)$. When Y_F is first reached, the follower invests and the leader's expected flow payoff is altered. Since value functions are forward-looking, $L_1(Y)$ anticipates the effect of the follower's action and must therefore meet $L_2(Y)$ at Y_F. Hence, a value matching condition holds at this point; however there is no optimality on the part of the leader, and so no corresponding smooth pasting condition. This implies

$$\frac{Y\pi(2,1)}{r-\mu}+B_{L1}Y^{\beta}-I=\frac{Y\pi(2,2)}{r-\mu}-I$$

that gives

$$B_{L1}=\frac{Y_F^{1-\beta}[\pi(2,2)-\pi(2,1)]}{r-\mu}$$

The usual value matching and smooth pasting conditions at the optimally chosen Y_L determine the other unknown variables:

$$\begin{cases}\dfrac{Y_L\pi(1,1)}{r-\mu}+B_{L0}^{\beta}Y_L=\dfrac{Y_L\pi(2,1)}{r-\mu}+B_{L1}^{\beta}Y_L-I\\ \dfrac{\pi(1,1)}{r-\mu}+\beta B_{L0}^{\beta-1}Y_L=\dfrac{\pi(2,1)}{r-\mu}+\beta B_{L1}^{\beta-1}Y_L\end{cases}$$

Solving the system, we can compute the value of the optimal trigger point Y_L (equation (10) in the text) and unknown constant B_{L0}, i.e.

$$B_{L0}=\left[\frac{Y_L[\pi(2,1)-\pi(1,1)]}{r-\mu}-I\right]Y_L^{\beta}+\frac{Y_F[\pi(2,2)-\pi(2,1)]}{r-\mu}Y_F^{\beta}$$

By simple substitution we get eq. (12) in the text. Let us now compute the value function of both firms in case of simultaneous adoption. Denoting by $F_S(Y)$ the option to invest of each firm, the optimal investment rule is given by:

$$F_S(Y)=Max\left\{V_S(Y)-\frac{Y\pi(1,1)}{r-\mu},\frac{1}{1+r}E[F_S(Y+dY|Y)]\right\}$$

where $V_S(Y)$ is the equity value of each firm net of the investment cost I, and E denotes the expectation. The value of the option to invest is

$$F_S(Y)=B_SY^{\beta}$$

where $\beta>1$ is the positive root of the usual characteristic equation and $B_S\geq 0$ is an unknown constant. The option value of waiting is

(A1) $$S_1(Y)=F_S(Y)+\frac{Y\pi(1,1)}{r-\mu}$$

We next consider the value of the firms in the stopping region, for $t \geq T_S$. The expected value of the firms equity must satisfy the following equation:

$$V_S(Y) = E\left[\int_t^{+\infty}[Y\pi(2,2)e^{-r(t-\tau)} - I]d\tau\right]$$

Working out the expectation, the above expression reduces to:

$$V_S(Y) = \frac{Y\pi(2,2)}{r-\mu} - I \tag{A2}$$

Putting togheter eqs. (A1) and (A2), we obtain eq. (13) in the text. Applying the standard value matching and smooth pasting conditions, it is possible to determine the critical value of the stochastic process Y_S that triggers the investment:

$$\begin{cases} F_S(Y_S) = V_S(Y_S) \\ F_S(Y_S) = V_S(Y_S) \end{cases}$$

By simple substitution, we get the trigger point Y_S (eq. (14) in the text) and the coefficient B_S:

$$B_S = \frac{[\pi(2,2) - \pi(1,1)]}{r-\mu}\frac{1}{\beta}Y_S^{1-\beta}$$

Finally, the necessary and sufficient condition for a simultaneous investment to occur in equilibrium is that $S(Y) \geq L(Y)$ for all $Y \in [Y_S, Y_L]$. By the convexity of the value functions, this requires that $S(Y) \geq L(Y)$ for all $Y \in (0, Y_S]$, i.e. $B_S \geq B_{L0}$. Therefore, $\frac{Y_S^{1-\beta}[\pi(2,2)-\pi(1,1)]}{r-\mu}\frac{1}{\beta} \geq \left[\frac{Y_L[\pi(2,1)-\pi(1,1)]}{r-\mu} - I\right]Y_L^{-\beta} + \frac{Y_F^{1-\beta}[\pi(2,2)-\pi(2,1)]}{r-\mu}$. Substituting the trigger points, Y_S, Y_L, Y_F, yields the result (eq. (15) in the text).

9. Appendix 2

Proof of Proposition 6. Let us define the function $\Delta(Y) = L_1(Y) - F_1(Y)$, describing the gain of pre-emption, where L_1(Y) is conditional on the «preemptor» having invested, and $F_1(Y)$ is the option value of the follower. By simple substitution we obtain

$$\Delta(Y) = \begin{array}{l} \frac{Y\pi(2,1)}{r-\mu} + \frac{Y_F(\pi(2,2)-\pi(2,1))}{r-\mu}\left(\frac{Y}{Y_F}\right)^{\beta} \\ -I - \frac{Y\pi(1,2)}{r-\mu} - \frac{\pi(2,2)-\pi(2,1)}{r-\mu}\frac{1}{\beta}Y_F\left(\frac{Y}{Y_F}\right)^{\beta} \end{array}$$

Now, to prove the existence and uniqueness of the trigger point Y_F, notice that:

1. $\Delta(0) = -I < 0$;
2. $\Delta(Y_F) = 0$.
3. By computing the first derivative $\Delta'(Y)$, and evaluating it at Y_F and 0, we get:

$$\Delta'(Y) = \begin{array}{l} \frac{\pi(2,1)}{r-\mu} + \frac{Y_F(\pi(2,2)-\pi(2,1))}{r-\mu} Y_F^{-\beta} \beta Y^{\beta-1} \\ -\frac{\pi(1,2)}{r-\mu} - \frac{\pi(2,2)-\pi(2,1)}{r-\mu} \frac{1}{\beta} Y_F^{1-\beta} \beta Y^{\beta-1} \end{array}$$

and thus

$$\Delta'(0) = \frac{\pi(2,1)-\pi(1,2)}{r-\mu} \gtrless 0$$

and

$$\Delta'(Y_F) = \frac{1}{r-\mu} + [(\pi(2,2) - \pi(2,1))(\beta - 1)] > 0.$$

The second derivative ensures now the monotonicity of the function, and therefore the uniqueness of the trigger point.

Proof of Proposition 7. In order to show the existence of both equilibria, we have to prove that $F_S(Y) > F_F(Y)$ for $Y \in (0, Y_F]$ and $F_S(Y) > V_F(Y)$ for all $Y \in (Y_F, Y_S]$. Let us define $D(Y) = F_S(Y) - V_S(Y) - I$, i.e. $D(Y) = \frac{1}{r-\mu}\frac{1}{\beta}[\pi(2,2) - \pi(1,1)]Y_S^{1-\beta}Y^{\beta} + \frac{Y\pi(1,1)}{r-\mu} - \frac{Y\pi(2,2)}{r-\mu} + I$. Note that $D(0) = I > 0$ and $D(Y_S) = 0$. Let us now compute $D'(Y) = \frac{1}{r-\mu}[\pi(2,2) - \pi(1,1)]$ $D' Y_S^{1-\beta} Y^{\beta-1} + \frac{\pi(1,1)}{r-\mu} - \frac{\pi(2,2)}{r-\mu}$. Note now that $D'(0) = -\frac{\pi(2,2)-\pi(1,1)}{r-\mu} < 0$ and $D'(Y_S) = 0$ and, finally, $D''(Y) = \frac{1}{r-\mu}(\pi(2,2) - \pi(1,1))Y_S^{1-\beta}(\beta - 1)Y^{\beta-2} > 0$
As $V_S(Y) \equiv V_F(Y)$ for $Y > Y_F$ and $Y_S > Y_F$, it follows that $D(Y_F) > 0$ and therefore $F_S(Y) > V_F(Y) - I$ for all $Y \in [Y_F, Y_S]$. Moreover, $F_S(0) > F_F(0)$ and $F_S(Y_F) > F_F(Y_F)$. By monotonicity of the functions $F_S(Y)$ and $F_F(Y)$ we get the result. ■

10. Appendix 3

Proof of Proposition 8. The cooperative trigger points are determined by the value matching and smooth pasting conditions at both points. At Y_{1L}, the conditions are the following:

$$\begin{cases} \dfrac{2Y_{1L}\pi(1,1)}{r-\mu} + B_0^\beta Y_{1L} + B_1^\beta Y_{1L} = \dfrac{2Y_{1L}\pi(2,1)}{r-\mu} + B_2^\beta Y_{1L} - I + B_3^\beta Y_{1L} \\ \dfrac{2\pi(1,1)}{r-\mu} + \beta B_0^{\beta-1} Y_{1L} + \beta B_1^{\beta-1} Y_{1L} = \dfrac{2\pi(2,1)}{r-\mu} + \beta B_2^{\beta-1} Y_{1L} + \beta B_3^{\beta-1} Y_{1L} \end{cases}$$

Solving the above system, we get the leader's trigger point, Y_{1L} (eq. (17) in the text). At Y_{2F} the conditions are the following:

$$\begin{cases} \dfrac{2Y_{2F}\pi(2,1)}{r-\mu} + B_2^\beta Y_{2F} - I + B_3^\beta Y_{2F} = \dfrac{2Y_{2F}\pi(2,2)}{r-\mu} - 2I \\ \dfrac{2\pi(2,1)}{r-\mu} + \beta B_2^{\beta-1} Y_{2F} + \beta B_3^{\beta-1} Y_{2F} = \dfrac{2\pi(2,2)}{r-\mu} \end{cases}$$

Again, solving this system we first get the follower's trigger point (eq. (18) in the text) and then

$$B_0 + B_1 = \frac{I}{\beta-1} Y_{1L}^{-\beta} + \frac{I(2-\beta)}{\beta-1} Y_{2L}^{-\beta}$$

Proof of Proposition 10. The necessary and sufficient condition for simultaneous investment to occur in equilibrium is $C_S(Y) > C_{LUF}(Y)$ for all $Y \in [Y_S, Y_L]$. The strict convexity of the value functions requires $C_S(Y) > C_{LUF}\ (Y)$ for all $Y \in [0,\ Y_S]$, i.e. $B_4 > B_0 + B_1$. Simple substitution yields to (25) in the text. ■

References

Bessen J. - Maskin E. (2009), Sequential Innovation, Patents, and Imitation, *RAND Journal of Economics*, vol. 40, no. 4, pp. 611-35.

Boyer M. - Gravel E. - Laserre P. (2004), Real Options and Strategic Competitions: A Survey, Working Paper of Université de Montreal, *mimeo*.

Bresnahan T.F. (1998), The Changing Structure of Innovation in Computing: Sources of and Threats to the Dominant US Position, in Mowery D. *et al.* (eds.), *America's Industrial Resurgence*.

Dasgupta P. (2003), Social Capital and Economic Performance: Analytics, in Ostrom E. - Ahn T.K. (eds.), *Foundations of Social Capital*, Cheltenham, UK, Edward Elgar.

Dixit A. (1993), The Art of Smooth Pasting, vol. 55, in Lesourne J. - Sonnenschein H. (eds.), *Fundamentals of Pure and Applied Economics*, Chur, Switzerland, Harwood Academic Publishers.

Dixit A. - Pindyck R.S. (1994), *Investment under Uncertainty*, Princeton, Princeton University Press.

Dutta P.K. - Rustichini A. (1993), A Theory of Stopping Time Game with Applications to Product Innovation and Asset Sales, *Economic Theory*, vol. 3, no. 4, pp. 743-763.

Fudenberg D. - Tirole J. (1985), Pre-emption and Rent Equalization in the Adoption of New Technology, *Review of Economic Studies*, vol. LII, no. 3, pp. 383-401.
Grenadier S.R. (1996), The Stategic Exercise of Options: Development Cascades and Overbuilding in Real Estate Markets, *The Journal of Finance*, vol. 51, no. 5, pp. 1653-1679.
Huisman K.J. - Kort P. (1999), Effects of Strategic Interactions on the Option Value of Waiting, CentER Discussion Paper N. 9992, Tilburg University, *mimeo*.
Huisman K.J. - Kort P. (2004), Strategic Technology Adoption Taking Into Account Future Technological Improvements: A Real Option Approach, *European Journal of Operational Research*, vol. 159, no. 3, pp. 705-728.
Kong J.J. - Kwok Y.K. (2007), Real Options in Strategic Investment Games between Two Asymmetric Firms, *European Journal of Operational Research*, n. 181, pp. 967-985.
Marseguerra G. - Cortelezzi F. - Dominioni A. (2006), Investment Timing Decisions in Stochastic Duopoly Model, *Chaos, Solitons & Fractals*, vol. 29, no. 3, pp. 611-625.
Marseguerra G. - Cortelezzi F. (2006), Investimenti irreversibili, interazione strategica e opzioni reali: analisi teorica ed applicazioni, *Economia Politica*, vol. XXIII, no. 2, pp. 255-292.
— (2009), Debt Financing and Real Estate Investment Timing Decisions, *Journal of Property Research*, vol. 26, no. 3, pp. 193-212.
Mason R. - Weeds H. (2010), Investment, Uncertainty and Pre-emption, *International Journal of Industrial Organization*, vol. 28, no. 3, pp. 278-287.
McDonald R. - Siegel D. (1984), The Value of Waiting to Invest, *Quarterly Journal of Economics*, vol. 101, no. 4, pp. 707-728.
Nocco A. (2007), Geographically Localized Externalities and Comparative Advantage in New Economic Geography, *Economia Politica*, vol. XXIV, no. 1, pp. 13-42.
Smets F. (1991), Exporting versus FDI: The Effect of Uncertainty, Irreversibilities and Strategic Interactions, Working Paper, Yale University, *mimeo*.
Trigeorgis L. (1996), *Real Options*, Cambridge, MA, MIT Press.
Varian H., (2001), High-Technology Industries and Market Structure, Working Paper, University of California, Berkeley, *mimeo*.
Weeds H. (2002), Strategic Delay in a Real Options Model of R&D Competition, *Review of Economic Studies*, vol. 69, no. 3, pp. 729-747.

Summary: Strategic Investment Timing Under Profit Complementarities (J.E.L. C73, D81, L13, O33)

This paper analyses strategic investment games between two firms that compete for the adoption of a new more efficient technology whose returns are uncertain. We assume that once one of the two firms adopted the new technology, joint adoption is preferable for both firms, that is there are profit complementarities in the product market. There are, moreover, externalities derivig from the first firm's investment. By modelling the switch from a well established technology to a new one as a dynamic stochastic game, we fully characterize the equilibria of the game under both non-cooperative and cooperative firms' behaviour. We show that in the cooperative equilibrium firms will invest later under negative externalities and earlier under positive externalities. Thus we identify circumstances in which competiton can be suboptimal (too much waiting). Overall, compared to earlier models that only allow for a new market game, our model examines a richer set of strategic interactions of adoption decisions.