

Q-switching and harmonic modelocking pulse instabilities of solid-state lasers

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Abstract. Passively modelocked solid-state lasers can exhibit two types of instabilities with very different origins. Near threshold, pulses are prone to the Q-switching instability, where pulse energy shows strong periodic modulation over successive roundtrips. This behaviour disappears as the pump power increases, giving way to the fundamental modelocked state—characterized by a single stable pulse circulating in the cavity. At higher pump levels, this state can become unstable again, leading to the generation of multiple equidistant pulses per roundtrip, forming harmonic modelocking states with 2, 3, ..., n pulses. These instabilities critically affect laser performance, especially in systems using slow saturable absorbers, where accurate modelling becomes particularly challenging. Despite their practical relevance, analytical expressions for the boundaries of these instability regimes are scarce. In this work, we derive such expressions from a recently proposed generalization of the Haus master equation, providing a compact framework to describe the onset of both Q-switching (QML) and harmonic modelocking (HML) in passively modelocked solid-state lasers. These results contribute to a deeper understanding of the dynamics involved and offer valuable guidance for experimental design and optimization.

Given the profound impact of passively modelocked solid-state lasers across science, technology, and industry [1], a theoretical understanding of the instabilities affecting modelocked pulses, including analytical expressions for the boundaries, is essential. This is particularly challenging with slow saturable absorbers, such as SESAM and carbon nanotubes, whose recovery time is comparable to or longer than the pulse duration. The key issue is the absence of a sufficiently simple theory to assess the dynamical behaviour of pulses. For fast absorbers, Haus' master equation (ME) provides such a theory, at least near threshold. However, for slow absorbers, Haus' ME is incomplete, struggling to incorporate the widely different timescales of material dynamics, especially the gain.

Classic theoretical approaches to these instabilities were developed by Haus [2] and expanded by Keller's group [3,4], using a rate equation model for average pulse power, gain, and saturable absorption. However, this model neglects finite gain bandwidth effects, as

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Haus' ME cannot account for them in dynamic regimes, particularly with slow saturable absorbers.

Using an approach successfully applied to active modelocking [5], we derived a master equation that consistently incorporates material dynamics across all relevant timescales while retaining gain bandwidth [6]. This novel ME theory has enabled us to obtain analytical expressions describing the boundaries of QML and the sequence of HML instabilities affecting solid-state lasers, which are omitted here for brevity. Examples of these behaviours can be found in Figs. 1 and 2, which are closely described by the analytical expressions.

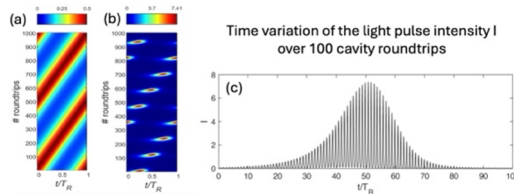


Fig. 1. Near the modelocking threshold, a single CW pulse circulates within the cavity (a). A slight increase in the pump power (b) triggers the QML instability, modulating the pulse height over 100 roundtrips (c), corresponding to one “isola” in (b). (a) and (b) are light intensity (I) density plots in the ME representation, with consecutive roundtrips (of duration T_R) stacked sequentially.

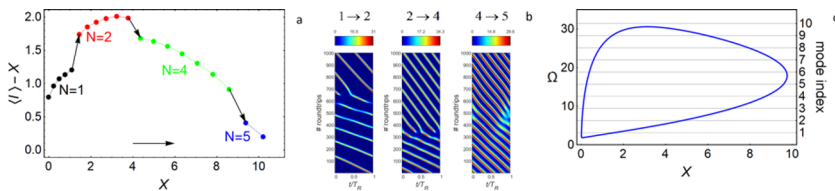


Fig. 2. HML transition sequence (from 1 to 5 pulses) observed as the pump X increases (a and b). Panel c shows the self-starting modelocking boundary in the pump(X) - sidemode frequency(Ω) plane. The crossings of the horizontal lines (sidemode frequencies) with the closed balloon in panel c mark the HML bifurcations, which closely align with the numerically observed transitions in panel a.

These results represent a significant advancement over previous treatments by incorporating finite bandwidth effects and relevant material timescales, which are crucial for accurately determining the stability of modelocked pulses.

This work is part of Project PID2023-153363NB-C22, funded by MCIU/AEI/10.13039/501100011033.

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