



Heterogeneity in rent-seeking contests with multiple stages: Theory and experimental evidence[☆]

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ABSTRACT

We investigate how heterogeneity in contestants' investment costs affects competition expenditures in a dynamic elimination contest with different seeding variants of contestants. Theory predicts that expenditures in dynamic contests are lower when competitors are heterogeneous than when they are homogeneous. Cost heterogeneity influences expenditures directly – by inducing weak and strong competitors to reduce their expenditures – and indirectly – through their influence on continuation values. We present evidence from lab experiments that is qualitatively in line with the theoretical prediction for contestants with low investment costs: they incorporate the heterogeneity and the differences in continuation values when competing in stage one and they decrease their expenditures when competing against a weak agent in stage two. For high-cost contestants, the theoretical predictions are not confirmed: expenditures in heterogeneous interactions are not lower and sometimes even higher. As a consequence, we find that total expenditures in heterogeneous dynamic contests are not necessarily lower than in homogeneous ones.

1. Introduction

Settings with several competitors expending resources to earn a prize (or rent) are ubiquitous. Applications include sports tournaments with play-offs, lobbying competitions to influence regulations or laws, competition for procurement contracts, political elections, or promotions in organizations, among many others. Two key features that characterize such contest settings in the real world are heterogeneity – contestants typically differ in dimensions relevant for competitive decisions – and dynamics – contests typically involve several stages. In sports tournaments, heterogeneous players compete over a number of rounds in play-offs, legislation typically involves several stages at which heterogeneous lobby groups compete, procurement contracts typically have a qualification stage, political elections like in the U.S. involve primaries, and promotions occur in multi-layered organizations.

Given the ubiquity of situations where agents spend resources in competing for a prize, it does not come as a surprise that the optimal design of contests is studied in different disciplines. However, theoretical as well as experimental studies that analyze explicitly the effect of heterogeneity on competition intensity in multi-stage tournaments are scarce. The existing theoretical literature either assumes

homogeneous contestants (Fu & Lu, 2012; Gradstein & Konrad, 1999; Stein & Rapoport, 2005), considers all-pay auction frameworks with more than two heterogeneous types (Groh et al., 2012), or focuses on other issues than the explicit effect of the degree of heterogeneity on expenditures (Kräkel, 2014; Ryvkin, 2011; Stein & Rapoport, 2004; Stracke, 2013; Stracke et al., 2015). Experimental studies of contests with heterogeneous agents typically focus on static contests (Anderson & Stafford, 2003; Bull et al., 1987; Kimbrough et al., 2014; Schotter & Weigelt, 1992).¹ While dynamic contests have been studied extensively, the focus was either again on homogeneous setups (Altmann et al., 2012; Deck & Kimbrough, 2015; Sheremeta, 2010b), or the design differs in other important dimensions from our setup – (Sheremeta, 2010a) allows to carry over endowment across stages, while (Stracke et al., 2014) analyze different prize structures and the effect on effort provision. Brown and Minor (2014) investigate spillover effects on effort of past and future competitions between heterogeneous agents in two-stage elimination contests and find evidence in support of their model predictions on winning probabilities using tennis data, but have no evidence on the effects on effort.

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¹ See Dechenaux et al. (2015) for an excellent survey of the experimental contest literature.

In this paper, we contribute to the literature by investigating how heterogeneity in contestants' investment costs affects individual and total expenditures in a dynamic elimination contest – both theoretically and experimentally. In our analysis, we focus on a two-stage version of the standard (Tullock, 1980) lottery contest with four agents. Within this framework, we compare expenditures in homogeneous settings, in which all agents have the same costs, to the expenditures in heterogeneous settings where agents have different costs in order to investigate how expenditures in the two stages of the contests are affected by the heterogeneity of contestants.

Concretely, our analysis considers the role of heterogeneity by investigating the consequences of the composition of the contestants in terms of different combinations of high- (H) and low- (L) cost contestants. We derive theoretical predictions about the effect of heterogeneity on individual expenditures in different stages as well as on the overall effect of heterogeneity on expenditures. Standard theory – based on the assumption of common knowledge of rationality and risk neutrality – predicts that aggregate expenditures of the group are lower on average if contestants are heterogeneous than if they are homogeneous. The reason is that competition is less intense if contestants are heterogeneous rather than homogeneous. The effect is driven by three underlying mechanisms: (i) high-cost agents reduce their expenditures in heterogeneous settings compared to homogeneous settings, because it is more costly for them to compete against a low-cost contestant; (ii) low-cost agents reduce their expenditures as well, as it is easier for them to win against a high-cost opponent; and (iii) first-stage expenditures are also influenced by the “continuation value” of the contest, which varies depending on the expected opponent in the second stage. In particular, each type would prefer to compete with a high-cost than with a low-cost opponent in the second stage.

We then test the theoretical predictions in a laboratory experiment. In the homogeneous cases, we investigate one treatment with only high-cost agents – HHHH – and one with only low-cost contestants – LLLL. In the heterogeneous settings, we focus on an equal number of low- and high-cost contestants and investigate two different seeding variants: While equal types compete in the pairwise interaction of the first stage in treatment LLHH, contestants in treatment LHLH are seeded in a way such that different types interact in the first stage. This design has the advantage of allowing us to study the role of heterogeneity in comparison to homogeneous settings with the same pool of contestants. This implies that we can compare the overall investments across all settings without having to account for differences in investment costs.

Our empirical results only partly confirm the theoretical predictions. Specifically, the qualitative prediction derived from our model are largely confirmed for the homogeneous contests — and for the heterogeneous contests they are largely confirmed for the low-cost agents, but not for the high-cost ones: High-cost agents do not lower stage-one expenditures in both seeding variants of the heterogeneous contest (LHLH and LLHH) compared to the homogeneous treatment HHHH. Similar results are obtained for the second stage — high-cost agents do not lower expenditures in heterogeneous stage-two interactions, compared to the homogeneous benchmark and sometimes even increase them. As a consequence total expenditures are not lower in heterogeneous than in homogeneous dynamic contests — contrary to the theoretical prediction.

We end our investigation by discussing potential explanations for these findings. They include, in particular, emotions associated with winning, particularly against a competitor that has a cost advantage. While broadly in line with such emotions, our results also indicate history dependence and point towards directions for future research.

The remainder of this paper is organized as follows: Section 2 analyzes the effects of variations in the level of investment costs and of cost-heterogeneity on the expenditures in a simple dynamic contest model. Section 3 outlines the experimental design and Section 4 states our main hypotheses. The experimental results are presented and discussed in Section 5, and Section 6 concludes.

2. Theoretical analysis

Consider the simplest version of a dynamic pairwise elimination contest with two stages, as depicted in Fig. 1. Each of the pairwise interactions is modeled as a (Tullock, 1980) lottery contest with linear investment costs. The four risk-neutral contestants compete for an indivisible rent of size R . The contestants are of two different types. Types differ in the cost per unit invested. The cost is c_L for low-cost contestants and c_H for high-cost contestants, with $c_H > c_L$. Types are common knowledge among contestants.

We consider two homogeneous and two heterogeneous versions of the contest. In the two homogeneous contests, LLLL and HHHH, the investment cost parameter is the same for all participants. In the heterogeneous versions, we assume that equal shares of the two types participate in the contest. This allows for two strategically different seeding variants: Contestants can either be seeded to ensure that equal types interact in the first stage (LLHH), or in such a way that both first-stage interactions are heterogeneous (LHLH).

The relevant equilibrium concept is Subgame Perfect Nash in all cases: we solve the game via backwards induction and consider the subgame of the second stage, before we determine equilibrium behavior in the first stage. Each subgame is a pairwise interaction between two contestants i and j with investment costs c_i and $c_j \in \{c_H, c_L\}$, respectively. Each contestant chooses her investment level to maximize the expected payoff in the respective stage. Contest investments increase the probability of receiving the expected payoff, but lead to costs that are independent of success or failure.

2.1. Stage two

In the second stage, agents compete for the indivisible rent R . The formal optimization problem of contestant i reads

$$\max_{x_{2i} \geq 0} \Pi_{2i}(x_{2i}, x_{2j}) = \frac{x_{2i}}{x_{2i} + x_{2j}} R - c_i x_{2i},$$

where x_{2i} (x_{2j}) denotes contest investment by i (j) in stage two. Following (Nti, 1999), first-order optimality conditions determine the unique pure strategy Nash equilibrium. Equilibrium investment levels satisfy

$$x_{2i}^* = \frac{c_j}{(c_i + c_j)^2} R; \quad x_{2j}^* = \frac{c_i}{(c_i + c_j)^2} R.$$

Therefore, equilibrium expenditures are

$$\begin{aligned} E_{2i}^* &= \frac{1}{4} R && \text{in the homogeneous settings, and} \\ E_{2i}^* &= \frac{c_i \cdot c_j}{(c_i + c_j)^2} R && \text{in the heterogeneous settings.} \end{aligned} \tag{1}$$

Inserting these equilibrium expenditures in the objective functions delivers the expected equilibrium payoffs:

$$\Pi_{2i}^* \equiv \Pi_{2i}(x_{2i}^*, x_{2j}^*) = \frac{c_j^2}{(c_i + c_j)^2} R. \tag{2}$$

Expected equilibrium payoffs are strictly positive, decrease in the own cost parameter, and increase in the cost parameter of the opponent.

2.2. Stage one

In stage one, contestants compete for a continuation value, that is, for the right to participate in the second stage. The continuation value is determined by the expected stage-two equilibrium payoff in Eq. (2). Consider the stage-one optimization problem of contestant i with investment costs c_i , who competes with contestant j for a given continuation value CV_j . The problem is formally defined as follows:

$$\max_{x_{1i} \geq 0} \Pi_{1i}(x_{1i}, x_{1j}) = \frac{x_{1i}}{x_{1i} + x_{1j}} CV_j - c_i x_{1i}, \tag{3}$$

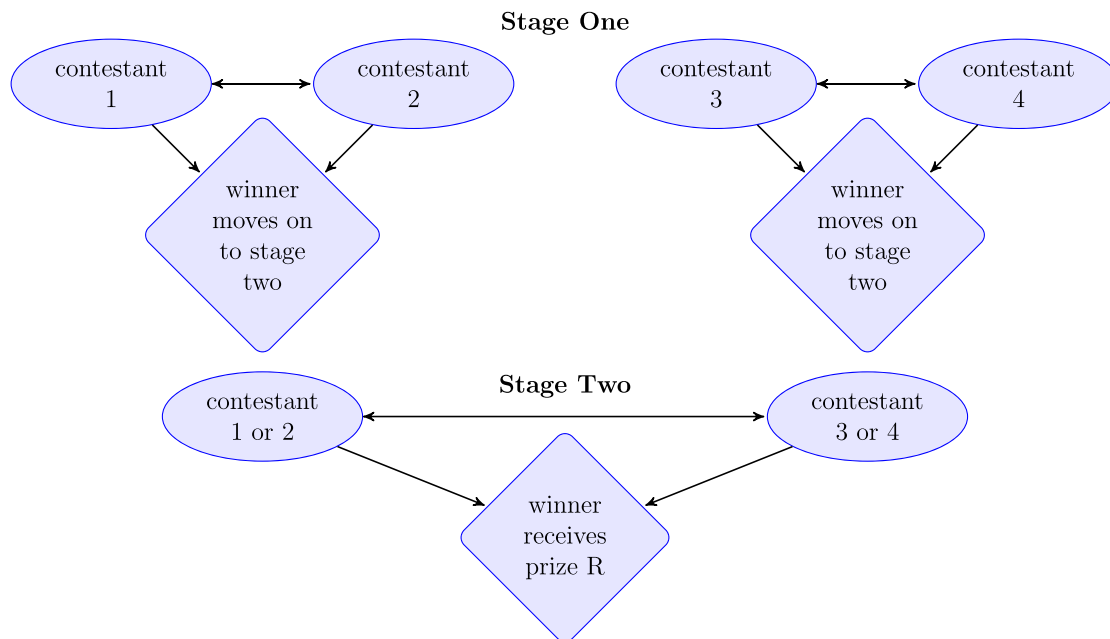


Fig. 1. Structure of the Contest.

where x_{1i} (x_{1j}) denotes contest investment by contestant i (j) in stage one. For given continuation values the first order conditions for Eq. (3) are

$$x_{1j} \cdot CV_i - c_i(x_{1i} + x_{1j})^2 = 0 \quad \text{and}$$

$$x_{1i} \cdot CV_j - c_j(x_{1i} + x_{1j})^2 = 0.$$

Hence, stage-one equilibrium investment by contestant i for given continuation value CV_i reads

$$x_{1i}^* = \frac{c_j CV_i CV_j}{(c_i CV_j + c_j CV_i)^2} CV_i. \tag{4}$$

The continuation values are affected by the seeding of types in stage one. Thus, we derive the explicit expressions for equilibrium investments and payoffs separately for the homogeneous settings LLLL and HHHH, and the heterogeneous settings LLHH and LHLH.

2.2.1. Homogeneous settings

In case of homogeneity, the continuation value is the same for all contestants and is independent of the level of unit costs in the unique equilibrium, which is symmetric.² Given $c_i = c_j$, it follows from (2) that the continuation value reads

$$CV = \frac{1}{4} R.$$

Inserting this expression in (4) gives the following equilibrium investment

$$x_{1i}^* = \frac{1}{16c_i} R,$$

where investment costs of all contestants could be either high or low. Hence, equilibrium expenditure in the homogeneous case is

$$E_{1i}^* = \frac{1}{16} R. \tag{5}$$

That is, individual expenditure is independent of the cost parameter.

² Asymmetric equilibria do not exist in a lottery contest — see Cornes and Hartley (2005) for details.

2.2.2. Setting LLHH

If contestants are heterogeneous and seeded in such a way that equal types interact in stage one, one high- and one low-cost contestant will make it to stage two. Thus, contestants know which type they will face in stage two if they win in stage one. Thus, it follows from Eq. (2) that the continuation values read

$$CV_L = \frac{c_H^2}{(c_L + c_H)^2} R \quad \text{and} \quad CV_H = \frac{c_L^2}{(c_L + c_H)^2} R$$

for a low- and a high-cost contestant, respectively.³

Inserting continuation values in (4) delivers stage-one equilibrium investments

$$x_{1L}^*(LLHH) = \frac{c_H^2}{4c_L(c_L + c_H)^2} R \quad \text{and} \quad x_{1H}^*(LLHH) = \frac{c_L^2}{4c_H(c_L + c_H)^2} R. \tag{6}$$

These result in the following equilibrium expenditures:

$$E_{1L}^*(LLHH) = \frac{c_H^2}{4(c_L + c_H)^2} R \quad \text{and} \quad E_{1H}^*(LLHH) = \frac{c_L^2}{4(c_L + c_H)^2} R. \tag{7}$$

2.2.3. Setting LHLH

If contestants are heterogeneous and seeded in such a way that different types interact in stage one, contestants do not know the type of opponent they would face in stage two in case of success in stage one. There are three possible stage-two interactions: a homogeneous pairing consisting of two low-cost (LL) or two high-cost (HH) agents, or a heterogeneous pairing consisting of one low and one high-cost agent (LH). This complicates the continuation values, since the value of participation in the second stage depends on the expected type of the opponent.

To illustrate this finding, consider the stage-one interaction between a low-cost and a high-cost contestant with continuation values CV_L and CV_H , respectively. Without loss of generality, we assume that contestants i and k have low costs, whereas contestants j and l have high costs, and that the two pairwise stage-one interactions are between contestants i and j , and between contestants k and l , respectively.

³ The continuation value of each contestant is independent of the outcome in the parallel stage-one interaction, since only the type (and not the identity) of the stage-two opponent matters for the expected equilibrium payoff.

In each interaction both contestants choose their optimal stage-one investment, given equilibrium behavior in any potential stage-two interaction. From (5) and (6) it follows that these values are formally defined as

$$CV_L = q_L \cdot \frac{R}{4} + (1 - q_L) \cdot \frac{c_H^2}{(c_L + c_H)^2} R; \quad CV_H = q_L \cdot \frac{c_L^2}{(c_L + c_H)^2} R + (1 - q_L) \cdot \frac{R}{4}, \tag{8}$$

where q_L is the probability that the low-cost contestant wins in the stage-one interaction. This probability is determined by contest investments in the stage-one interaction, and since the continuation values in the parallel stage-one interaction have the same structure, stage-one investment choices by all four contestants are interdependent through endogenously determined continuation values. It can be shown, however, that the probability of a low-cost contestant to win in stage one is the same in both stage-one interactions in the unique equilibrium. This symmetry allows determining the equilibrium stage-one winning probability of a low-cost contestant, q_L^* (details are provided in Appendix A):

$$q_L^* = \frac{(c_H - c_L)(c_H + c_L)^2 + \sqrt{64c_H^3c_L^3 + (c_L - c_H)^2(c_L + c_H)^4}}{(c_H - c_L)(c_H + c_L)^2 + \sqrt{64c_H^3c_L^3 + (c_L - c_H)^2(c_L + c_H)^4} + 8c_L^3}. \tag{9}$$

Inserting Eq. (9) into Eq. (8) gives the following continuation values:

$$CV_i(x_{iL}^*, x_{iH}^*) = CV_k(x_{iL}^*, x_{iH}^*) = \frac{(c_L + c_H)^2 F^*(c_L, c_H) + 4c_H^2}{4(c_L + c_H)^2 [1 + F^*(c_L, c_H)]} R,$$

$$CV_j(x_{iL}^*, x_{iH}^*) = CV_l(x_{iL}^*, x_{iH}^*) = \frac{(c_L + c_H)^2 + 4c_L^2 F^*(c_L, c_H)}{4(c_L + c_H)^2 [1 + F^*(c_L, c_H)]} R,$$

where $F^*(c_L, c_H)$ is given by

$$F^*(c_L, c_H) = \frac{(c_H - c_L)(c_L + c_H)^2 + \sqrt{64c_H^3c_L^3 + (c_L - c_H)^2(c_L + c_H)^4}}{8c_L^3},$$

as derived in Eq. (15) in Appendix A. Note that $CV_i(x_{iL}^*, x_{iH}^*) = CV_k(x_{iL}^*, x_{iH}^*)$ and $CV_j(x_{iL}^*, x_{iH}^*) = CV_l(x_{iL}^*, x_{iH}^*)$ due to symmetry. Given these continuation values, stage one equilibrium investments can be determined as

$$x_{iL}^*(LHLH) \equiv x_{iL}^*(LHLH) = x_{iL}^*(LHLH)$$

$$= \frac{(c_L + c_H)^2 F^*(c_L, c_H) + 4c_H^2 F^*(c_L, c_H)}{4c_L(c_L + c_H)^2 [1 + F^*(c_L, c_H)]^3} R$$

$$x_{iH}^*(LHLH) \equiv x_{iH}^*(LHLH) = x_{iH}^*(LHLH)$$

$$= \frac{(c_L + c_H)^2 F^*(c_L, c_H) + 4c_L^2 F^*(c_L, c_H)^2}{4c_H(c_L + c_H)^2 [1 + F^*(c_L, c_H)]^3} R.$$

This results in equilibrium expenditures of

$$E_{iL}^*(LHLH) = \frac{(c_L + c_H)^2 F^*(c_L, c_H) + 4c_H^2 F^*(c_L, c_H)}{4(c_L + c_H)^2 [1 + F^*(c_L, c_H)]^3} R$$

$$E_{iH}^*(LHLH) = \frac{(c_L + c_H)^2 F^*(c_L, c_H) + 4c_L^2 F^*(c_L, c_H)^2}{4(c_L + c_H)^2 [1 + F^*(c_L, c_H)]^3} R. \tag{10}$$

2.3. Comparing expenditures

For the comparisons in this subsection we define TE_t as the sum of expenditures for stage t , that is

$$TE_1 = \sum E_{1i}$$

$$TE_2 = \sum E_{2i}.$$

Total expenditure across both stages TE is then defined as TE_1 plus TE_2 . From the equilibrium expenditures stated in Eqs. (1) and (5) we immediately get the following result:

Proposition 1 (Homogeneity). *Total expenditures across both stages do not depend on the level of the cost parameter in homogeneous settings. That is,*

$$TE(LLLL) = TE(HHHH) \equiv TE(HOM)$$

While this result has been documented in previous work (e.g., Stracke et al., 2014), it has not received much attention in the literature.

The equilibrium expenditures stated in Eq. (1), (5), (7) and (10) together imply:

Proposition 2 (Aggregate Effect of Heterogeneity). *Total expenditures across both stages are lower if contestants are heterogeneous than if they are homogeneous, and they are lower in LHLH than in LLHH. That is,*

$$TE(HOM) > TE(LLHH) > TE(LHLH)$$

Proof. See Appendix B.

Thus, theory predicts that aggregate expenditures are lower if contestants are heterogeneous than if they are homogeneous and that the negative impact of heterogeneity on total expenditures is more pronounced in LHLH than in LLHH. The reason for the former result is that competition is less intense if contestants are heterogeneous rather than homogeneous: high-cost agents reduce their expenditures in comparison to a homogeneous setting because it is more costly for them to compete against a low-cost contestant, and low-cost agents reduce their expenditures as a reaction to the reduction of expenditures of their opponent. Hence, the result established for static models that total expenditures are lower in heterogeneous than in homogeneous interactions carries over to dynamic structures (Anderson & Stafford, 2003). The intuition for the latter result (more pronounced impact of heterogeneity in LHLH than in LLHH) is that in LHLH the competition is already heterogeneous in stage one, while in LLHH the stage-one competition is homogeneous. This stage-one effect is stronger than the countervailing stage-two effect stemming from the fact that the stage-two interaction is always heterogeneous in LLHH, while it is sometimes homogeneous in LHLH.

3. Experimental design and procedures

Our experimental design involves four treatments that correspond to the four settings described in the previous section, with investment cost parameters c_L and c_H set to 1 and 2.5, respectively, and the contested rent R set to 240.⁴

The currency used in the experiment is Experimental Currency Units, ECUs. The set-up replicates the (Tullock, 1980) lottery contest technology studied in the theoretical analysis, and it was explained to subjects using a lottery analogy. Specifically, participants were told that they could buy a number of balls in each interaction for the price of $c_L = 1$ ECU or $c_H = 2.5$ ECUs, respectively. The balls purchased by the subjects as well as those purchased by their respective opponents were then said to be placed in the same ballot box, out of which one ball would be randomly drawn subsequently. To buy their desired number of balls, participants received an endowment of 240 ECUs at the beginning of stage one in each decision round. This endowment could then be used to buy balls in both stages, i.e., a subject that reached stage two could use whatever remained of the endowment from stage one to buy balls in stage two.

Subjects played 30 rounds of the game, and were randomly re-matched at the beginning of each round.⁵ Transfers across decision rounds were not possible and incorporating the endowments in the theoretical analysis does not affect the results since they drop out in the maximization. The part of the endowment that a participant did not use to buy balls was added to the payoffs for that round. At the end of the experiment, four rounds were randomly chosen and paid out.

⁴ The chosen amounts ensure existence and uniqueness of equilibrium in each of the four treatments despite the discrete grid on the strategy space.

⁵ After each decision round, participants were informed about their own decision, the decision(s) of their immediate opponent(s), and about their own payoff.

Table 1
Sessions per treatment and number of participants.

Treatment	# sessions	# rounds	# Participants
LLLL	5	30	100
LLHH	5	30	100
LHLH	5	30	100
HHHH	5	30	100

We ran a total of 20 sessions, which resulted in the number of observations as summarized in Table 1. In all our regressions, we control for round fixed effects to control for systematic variation across different rounds of the experiment. At the beginning of the experiment, subjects received instructions for the respective treatment, answered a set of control questions, and were informed about their cost parameter.⁶ The experiment was programmed in z-Tree (Fischbacher, 2007) and 200 ECUs corresponded to 1 Euro. Participants were students from the subject pool of the University of Innsbruck and were recruited via ORSEE (Greiner, 2004). Each session lasted approx. 70 minutes, and average earnings were approximately 11 Euros.

4. Hypotheses

The different treatment variants allow analyzing the reaction to heterogeneity in different stages of the contest. By comparing stage-one expenditures in the homogeneous treatments with those in treatment LLHH, we can test whether types rationally react to differences in continuation values resulting from homogeneous vs. heterogeneous interaction on stage two. In addition, a comparison of stage-two expenditures in treatment LLHH with those in the homogeneous benchmarks LLLL and HHHH allows us to test how types respond to heterogeneity in the direct interaction in stage two. In the following we list a number of hypotheses – derived from the theoretical framework in Section 2 – based on such comparisons.⁷ Table 2 shows the predicted equilibrium expenditures from Section 2 with the parameters introduced in the previous section.

4.1. Homogeneous: Individual expenditures

The following predictions regarding the two homogeneous treatments follow directly from Eq. (1), (2) and (5):

Hypothesis 1. Expenditures of low-cost and high-cost agents in homogeneous settings: Individual expenditures are the same in treatments LLLL and HHHH, both in stage one and in stage two:

$$(i) \quad E_{1L}^*(LLLL) = E_{1H}^*(HHHH)$$

$$(ii) \quad E_{2L}^*(LLLL) = E_{2H}^*(HHHH).$$

⁶ A translated version of the instructions is provided in the Online Appendix. The original (German) instructions are available from the authors upon request. In some of the sessions, we elicited additional information about respondents (such as responses about risk aversion) after the experimental treatments. Since this information is not available for all treatments and sessions, we do not include these variables as controls in our regression. However, due to the experimental randomization, this variation should not be related to any of our treatment effects. In fact, for the data we have, we find no significant heterogeneity in risk aversion across treatments.

⁷ Assuming a standard deviation of 25% of the absolute values of the estimated expenditures, and assuming that 50% of the individuals make it into stage two, the number of observations we collected (summarized in Table 1) make sure that we are able to identify the predicted differences in individual expenditures with an α of 5% and a power of 80%.

4.2. Heterogeneous: Individual expenditures in stage one

From Eq. (5), (7) and (10) we can derive the following predictions regarding the expenditures in heterogeneous as compared to homogeneous treatments.

Hypothesis 2. Expenditures of low-cost agents in stage one of heterogeneous settings: In stage one, the low-cost agent has higher expenditures in treatment LLHH as compared to the homogeneous treatment, lower expenditures in treatment LHLH as compared to the homogeneous treatment, and lower expenditures in treatment LHLH as compared to treatment LLHH:

$$(i) \quad E_{1L}^*(LLHH) > E_{1L}^*(LLLL)$$

$$(ii) \quad E_{1L}^*(LHLH) < E_{1L}^*(LLLL)$$

$$(iii) \quad E_{1L}^*(LHLH) < E_{1L}^*(LLHH)$$

Hypothesis 3. Expenditures of high-cost agents in stage one of heterogeneous settings: In stage one, the high-cost agent has lower expenditures in treatment LLHH as compared to the homogeneous treatment, lower expenditures in treatment LHLH as compared to the homogeneous treatment, and lower expenditures in treatment LHLH as compared to treatment LLHH:

$$(i) \quad E_{1H}^*(LLHH) < E_{1H}^*(HHHH)$$

$$(ii) \quad E_{1H}^*(LHLH) < E_{1H}^*(HHHH)$$

$$(iii) \quad E_{1H}^*(LHLH) < E_{1H}^*(LLHH)$$

To understand predictions H_2 and H_3 it is important to notice that the behavior of agents in the first stage does not only depend on the direct interaction, but also on the continuation value. Concretely, individual expenditures in stage one do not only depend on the respective agent's own cost parameter and on that of the direct competitor, but also on the type of the opponent the agent expects to meet in the second stage of the contest.

Let us consider *low-cost* agents first. Part (i) of hypothesis H_2 states that in stage one low-cost agents have an incentive to expend more in the homogeneous interaction LL of treatment LLHH than in the homogeneous interaction LL of treatment LLLL. This is due to the fact that in the former setting the continuation value is higher as low-cost agents anticipate that they will face a high-cost agent in stage two for sure, while in LLLL they know that they will face a low-cost agent in stage two for sure. The comparison between LHLH and LLLL is slightly more complicated: First, the stage-one interaction is heterogeneous in LHLH but homogeneous in LLLL, inducing lower expenditures in LHLH. Secondly, the continuation value is *higher* in LHLH than in LLLL, due to the fact that in the former there exists a positive chance of facing an opponent with high costs while there is no such chance in the latter. When combining these two effects, the first effect prevails, which results in lower expenditures in LHLH as compared to LLLL. Comparing treatment LHLH to treatment LLHH, both effects described above go into the same direction: the stage one interaction is heterogeneous in LHLH but homogeneous in LLLL, inducing lower expenditures in LHLH; and the continuation value is lower in LHLH than in LLHH, due to the fact that there exists a positive probability of facing an opponent with low costs in LHLH but not in LLHH. Thus, expenditures are lower in treatment LHLH.

In comparison to low-cost agents, *high-cost* agents have a clear incentive to reduce expenditures in stage one of both heterogeneous treatments — compared to the homogeneous benchmark: Since they face a low-cost agent in stage two of treatment LLHH, the continuation value is lower for them in treatment LLHH as compared to the homogeneous setting — leading to lower expenditures, as compared to the homogeneous treatment HHHH. Comparing HHHH to LHLH, the agent has a positive chance to meet an agent with low costs in stage two of the latter (while there is no such chance in the former), which reduces the

continuation value as compared to the homogeneous case; in addition, the stage one interaction is against a low-cost agent, again leading to lower expenditures in comparison to the homogeneous setting. Both effects go into the same direction, resulting in lower expenditures in LHLH as compared to HHHH. Comparing LLHH and LHLH we find, similar to the low-cost agents, a stronger effect of heterogeneity in the direct interaction, meaning that expenditures are lower in the stage one interaction of LHLH than in LLHH, although the continuation value is higher in LHLH than in LLHH.

4.3. Heterogeneous: Individual expenditures in stage two

In the second stage, agents only consider the type of agent they actually face in the pairwise interaction. Thus, heterogeneity is expected to lead to lower expenditures independent of the type of the agent, although, again, the motivation to lower expenditures is different: the favorite (low-cost agent) invests less due to the weaker opponent and the underdog (high-cost agent) invests less because he has a lower chance to win against a low-cost agent.

Hypothesis 4. Expenditures of low-cost agents in stage two of heterogeneous settings: In stage two, the low-cost agent has lower expenditures in treatment LLHH as compared to the homogeneous treatment, lower expenditures in LH-interactions of treatment LHLH as compared to the homogeneous treatment, and the same expenditures in LL-interactions of treatment LHLH as compared to the homogeneous treatment:

- (i) $E_{2L}^*(LH|LLHH) < E_{2L}^*(LLLL)$
- (ii) $E_{2L}^*(LH|LHLH) < E_{2L}^*(LLLL)$
- (iii) $E_{2L}^*(LL|LHLH) = E_{2L}^*(LLLL)$

Hypothesis 5. Expenditures of high-cost agents in stage two of heterogeneous settings: In stage two, the high-cost agent has lower expenditures in treatment LLHH as compared to the homogeneous treatment, lower expenditures in LH-interactions of treatment LHLH as compared to the homogeneous treatment, and the same expenditures in HH-interactions of treatment LHLH as compared to the homogeneous treatment:

- (i) $E_{2H}^*(LH|LLHH) < E_{2H}^*(HHHH)$
- (ii) $E_{2H}^*(LH|LHLH) < E_{2H}^*(HHHH)$
- (iii) $E_{2H}^*(HH|LHLH) = E_{2H}^*(HHHH)$

Note that heterogeneity is basically shifted across stages when moving from LLHH to LHLH. The two stage-one interactions are homogeneous in setting LLHH, while different types interact in stage one of setting LHLH. At the same time, the stage-two interaction is always heterogeneous in LLHH and sometimes homogeneous in LHLH.

4.4. Total expenditures

Hypothesis H_1 immediately implies:

Hypothesis 6. Total expenditures in homogeneous settings: Total expenditures across both stages are the same in the two homogeneous treatments LLLL and HHHH:

$$TE^*(LLLL) = TE^*(HHHH)$$

Proposition 2 states that total expenditures are lower if contestants are heterogeneous than if they are homogeneous and that heterogeneity has a more pronounced effect on total expenditures in LHLH than in LLHH. There are different ways to test the first part of this statement in our experiments. One way to test it is by merging the data from LLLL with that from HHHH to HOM and to then compare HOM to the data of the two heterogeneous treatments. The following prediction is based on these comparisons:

Table 2
Theoretical predictions for individual and total expenditures.

	Homogeneous Treatments		Heterogeneous Treatments			
	LLLL	HHHH	LLHH	LHLH		
Stage One						
E_{1L}	15.00		30.61	7.54		
E_{1H}		15.00	4.90	2.75		
Stage Two						
	LL	HH	LH	LL	LH	HH
E_{2L}	60.00		48.98	60.00	48.98	
E_{2H}		60.00	48.98		48.98	60.00
TE	180.00	180.00	168.98		135.67	

Hypothesis 7. The effect of heterogeneity on total expenditures: Total expenditures across both stages are higher in the homogeneous than in the heterogeneous treatments, and the expenditure-decreasing effect of heterogeneity is more pronounced in LHLH than in LLHH:

- (i) $TE^*(HOM) > TE^*(LLHH)$
- (ii) $TE^*(HOM) > TE^*(LHLH)$
- (iii) $TE^*(LLHH) > TE^*(LHLH)$

5. Experimental results

5.1. Overview

Table 3 summarizes the observed means for stage-one and stage-two expenditures by treatment. The empirically observed expenditures substantially exceed their theoretical counterparts in both stages in all treatments, as can be seen by comparing the actual means displayed in **Table 3** to the predictions displayed in **Table 2**. This reflects the well-known result of ‘overdissipation’ or ‘overbidding’ – expenditures being considerably larger than the theoretical benchmark — that has been documented in the existing literature (Altmann et al., 2012; Sheremeta, 2010b).⁸

Turning to the development over the different rounds, we find that overbidding decreases over rounds in the first, but not in the second stage of the contest (see Figs. 2–4)– a finding which is again in line with the literature (Sheremeta, 2010a). To account for these dynamics, which are unrelated to our research question, we conduct the analysis controlling for a full set of round fixed effects. Given the considerable difference between theoretical benchmarks and experimental results, in the subsequent analysis, we focus on testing the qualitative (rather than the quantitative) predictions of our hypotheses.

For the evaluation of treatment effects we conduct panel regressions that account for individual random effects, with standard errors clustered on the individual level. Since treatment effects are identified by between-subject variation, we cannot estimate models that account for individual fixed effects. Instead, we estimate a random effects specification to account for unobserved heterogeneity at the individual level. In addition, in the tables we report p-values of non-parametric Mann–Whitney-U tests, where one session is considered as one independent observation. The dependent variables are the expenditures of a given type in a given stage of the contest. We evaluate the main effect of a treatment or interaction with the treatment using a binary treatment variable.⁹ For clarity, we present the respective treatment comparisons one-by-one, instead of pooling all data in one regression.

⁸ Parco et al. (2005) and Amegashie et al. (2007) also observe relative over-dissipation in the first stage of a two-stage contest, but their results are less relevant for the qualification of our findings since in their experiments agents are budget constrained. Overbidding is not a particular feature of multi-stage contests, but can be observed in almost any contest experiment — see Sheremeta (2013) for an extensive survey and potential explanations of this behavior.

⁹ In addition to that, we include the estimation results of regressions on additional explanatory variables to check for robustness of our results. Those

Table 3
Average individual and total expenditures.

	Homogeneous Treatments		Heterogeneous Treatments			
	LLLL	HHHH	LLHH	LL	LHLH	HH
Stage One						
E_{1L}	34.33 (6.40)		46.21 (6.21)		26.75 (5.92)	
E_{1H}		33.14 (8.00)	28.02 (8.15)		29.06 (9.47)	
Stage Two						
E_{2L}	83.47 (11.20)		64.13 (7.03)	86.14 (16.87)	73.65 (14.50)	
E_{2H}		76.32 (6.53)	86.35 (10.78)		76.61 (15.92)	87.94 (6.54)
TE	305.13 (37.19)	286.92 (29.49)	303.54 (36.87)		274.77 (48.91)	

Standard deviations in parentheses. 1 Observation = 1 session, i.e. $N = 5$ in each cell.

Table 4
Random-effect panel regression of expenditures in HHHH and LLLL.

Dep. var:	(1)	(2)
	Stage one own expenditures	Stage Two own expenditures
LLLL	1.19 (0.73) [0.75]	6.68 (0.14) [0.25]
Observations	6000	3000

LLLL = 1 if treatment is LLLL and zero otherwise. All specifications contain a full set of round fixed effects. We report p-values based on standard errors clustered at the individual level in parentheses; p-values based on non-parametric Mann-Whitney-U tests ($N = 5$ in each treatment) are reported in squared brackets.

5.2. Homogeneous: Individual expenditures

Random-effect panel regressions of the expenditures in the homogeneous treatments (Table 4) indicate that individual expenditures on both stages are not significantly different across homogeneous treatments (first stage, LLLL: 34.33 vs. HHHH: 33.14, $p = 0.73$; second stage, LLLL: 83.47 vs. HHHH: 76.32, $p = 0.14$). These results are confirmed by two two-sample Wilcoxon Mann-Whitney-U (MWU) tests: first stage, $p = 0.75$; second stage, $p = 0.25$). This leads to the following result:

Result 1. Expenditures of low-cost and high-cost agents in homogeneous settings: In line with H_1 individual expenditures on both stages are not significantly different between LLLL and HHHH.

5.3. Heterogeneous: Individual expenditures in stage one

The stage-one expenditures of low-cost agents react in the predicted way to the differences in the continuation value between LLHH and LLLL: Low-cost agents have higher individual expenditures in LLHH, where they face a higher continuation value than in the homogeneous benchmark LLLL, although the competition is against a low-cost agent in both cases (individual expenditures of low-cost agents are 46.21 in LLHH vs. 34.33 in LLLL, $p = 0.03$, see reg. (1) in Table 5, and a two sample MWU-test, $p = 0.03$).¹⁰ As can be seen in panel (a) of Fig. 2, the predicted relation holds for all rounds.

additional variables are a time trend, lagged variables of the expenditures of the opponent in the two stages of the previous round, and an indicator whether the subject won in one of the stages of the previous round — see Tables A.1 and A.2 in the Online Appendix.

¹⁰ Table A.1 in the Online Appendix includes controls, and Tables A.3 and A.4 report the results for the first and the last 15 periods.

Part (ii) of prediction H_2 regards the more complicated comparison between LHLH and LLLL. In LHLH the current interaction is heterogeneous (implying incentives to decrease the expenditures) but the continuation value is higher (implying incentives to increase expenditures) and in theory the former effect is stronger than the latter — implying that expenditures should be lower in LHLH. And this is exactly what we find in the data (individual expenditures of low-cost agents are 26.75 in LHLH vs. 34.33 in LLLL; $p = 0.03$, see reg. (2) in Table 5, and a two sample MWU-test, $p = 0.05$). Again, this result holds over all rounds (see panel (a) of Fig. 2).

Turning to the comparison of the two heterogeneous treatments LLHH and LHLH where the heterogeneity effect (heterogeneity in the current interaction decreases expenditures of both agents) and the continuation value effect (individual expenditures increase in the continuation value) both point in the same direction — namely that individual expenditures of low-cost agents should be lower in LHLH — we indeed find that they are significantly lower (46.21 vs. 26.75, $p < 0.01$, see column (3) in Table 5, and a two sample MWU-test, $p < 0.01$). Again this relationship holds over all rounds (Fig. 2). We summarize these findings as:

Result 2. Expenditures of low-cost agents in stage one of heterogeneous settings: In line with prediction H_2 low-cost agents expend significantly more in treatment LLHH than in the homogeneous benchmark LLLL, significantly less in treatment LHLH than in the homogeneous benchmark LLLL, and significantly more in treatment LLHH than in treatment LHLH.

In contrast to the low-cost agents, high-cost agents have a clear incentive to reduce their stage-one expenditures in both heterogeneous treatments compared to the homogeneous benchmark. In the comparison LLHH vs. HHHH in part (i) of prediction H_3 , the predicted incentive to reduce expenditures is based purely on the continuation value effect (as the current interaction is homogeneous in both treatments). This effect predicts lower expenditures in LLHH where high-cost agents know that they will face a strong competitor for sure in stage two. In line with the prediction, high-cost agents indeed reduce their expenditures somewhat (from 33.14 in HHHH to 28.02 in LLHH), yet the difference is not significant ($p = 0.22$, see column (4) in Table 5, and a two sample MWU-test, $p = 0.46$). From panel (b) of Fig. 2 we can see that the qualitative relationship of expenditures being lower in treatment LLHH as compared to treatment HHHH holds for the last 15 rounds. This is confirmed by a panel regression of expenditures based only on the data of the last 15 rounds (see Table A.4 in the Online Appendix.¹¹ In the comparison LHLH vs. HHHH of part (ii) of prediction H_3 , the incentive

¹¹ When we include additional explanatory variables, expenditures are also lower in treatment LLHH as compared to HHHH (see column (4) of Table A.1

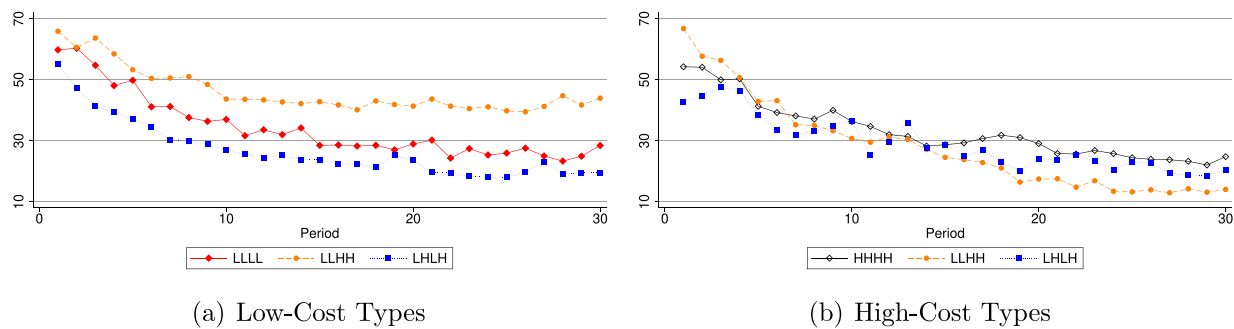


Fig. 2. Stage-One Expenditures by Cost-Type and Decision Round.

to reduce stage-one expenditures is even more pronounced: While the agent has a non-trivial chance to meet an agent with low costs in stage two of the former, there is no such chance in the latter. Thus, the continuation value is lower in the former. In addition, the stage-one interaction is against a low-cost agent in the former but against another high-cost agent in the latter. Thus, the heterogeneity effect also predicts lower expenditures in the former. Again, high-cost agents exhibit lower expenditures, but the difference is not significant (33.14 in HHHH vs. 29.06 in LHLH, $p = 0.39$, see column (5) in Table 5, and a two sample MWU-test, $p = 0.46$).

Turning to the comparison of the two heterogeneous treatments (where the continuation value effect and the heterogeneity effect point in different directions for high-cost agents) theory predicts that expenditures are lower in LHLH than in LLHH. In this contrast, the difference is small and stage-one expenditures of high-cost agents do not differ significantly across the two heterogeneous treatments, which is not in line with the prediction (they are 28.02 in LLHH vs. 29.06 in LHLH, $p = 0.84$, see column (6) in Table 5, and a two sample MWU-test, $p = 0.75$). The following statement summarizes these findings:

Result 3. Expenditures of high-cost agents in stage one of heterogeneous settings: In line with part (i) of H_3 , high-cost agents expend less in treatment LLHH than in the homogeneous benchmark. The difference is only significant for the second half of the experiment, however. In contrast to parts (ii) and (iii) of H_3 , high-cost agents do not expend significantly less in treatment LHLH as compared to the homogeneous benchmark HHHH, and they do not expend significantly less in treatment LHLH as compared to the treatment LLHH.

5.4. Heterogeneous: Individual expenditures in stage two

To investigate how high- and low-cost agents respond to heterogeneity in stage two, we compare stage-two expenditures of a given type in homogeneous and heterogeneous interactions. For stage two, theory predicts that expenditures only depend on the type of the opponent in the actual interaction; an opponent with different costs should lead to lower expenditures compared to an opponent of the same type.

In treatment LLHH, low-cost types face a high-cost agent in stage two. They are therefore predicted to expend less than in treatment LLLL, where they face a low-cost agent in stage two. In line with this prediction, they spend less in the LH interaction of LLHH compared to

in the Online Appendix: $p=0.03$). In addition, we find evidence for history dependence in the sense that stage-one expenditures of low-cost types are influenced by success on the first stage during the previous round of the experiment; stage-one expenditures are higher among low-cost and among high-cost types after winning the second stage during the previous round. Likewise, the expenditure of the respective opponent in the previous round of the experiment has an effect on stage-one expenditures.

the LL interaction of LLLL (64.13 vs. 83.47, $p < 0.01$, see column (1) in Table 6, and a two sample MWU-test, $p = 0.02$).¹² As can be seen in panel (a) of Fig. 3 this difference in expenditures holds for all rounds.¹³

Turning to the comparison of treatment LHLH to treatment LLLL, two interaction variants are possible for stage two of LHLH – LH and LL. For the LH interaction the prediction is again that expenditures are lower than in the homogeneous interaction LL of treatment LLLL. In line with this prediction low-cost agents reduce their expenditures somewhat (from 83.47 in LL|LLLL to 73.65 in LH|LHLH), the difference is not significant, however ($p = 0.14$, see column (2) in Table 6, and a two sample MWU-test, $p = 0.35$). Panel (b) of Fig. 3 suggests that the difference is more pronounced in the second half of the experiment. This is confirmed by a panel regression based only on the data of the last 15 rounds, although the effect turns out to be insignificant (see Table A.6 in the Online Appendix).

For the LL variant of the stage-two interaction in LHLH the prediction is that low-cost agents spend the same amount as in the LL variant of the stage-two interaction in LLLL. This is indeed the case in the data (86.14 vs. 83.47, $p = 0.79$, see column (3) in Table 6, and a two sample MWU-test, $p = 0.92$).

Result 4. Expenditures of low-cost agents in stage two of heterogeneous settings: In line with parts (i) and (ii) of prediction H_4 , stage-two expenditures of low-cost agents are lower in treatment LLHH than in treatment LLLL, and are lower in the LH interaction of treatment LHLH than in the LL interaction of treatment LLLL. While the former difference is highly significant over all rounds, the latter difference is significant only for the second half of the experiment. In line with part (iii) of prediction H_4 , stage-two expenditures of low-cost agents do not differ significantly between the LL interaction of treatment LHLH and the LL interaction of treatment LLLL.

Consider now stage-two expenditures of high-cost agents. They should spend less in the heterogeneous interactions LH than in the homogeneous interactions HH. They consistently violate this prediction. In the comparison of treatment LLHH to treatment HHHH, high-cost agents spend (weakly) more (although not significantly so) in the LH interaction of treatment LLHH than in the HH interaction of treatment HHHH (86.35 vs. 76.32, $p = 0.17$, see column (4) in Table 6, and a

¹² Table A.2 in the Online Appendix includes controls. Again, we also find evidence for history dependence in the sense that stage-two expenditures of both types are influenced by success on the first and second stage during the previous round of the experiment. Likewise, the expenditure of the respective opponent on the second stage during the previous round of the experiment has an effect on stage-two expenditures of low-cost types. Tables A.5 and A.6 report the results for the first and last 15 periods.

¹³ Also within treatment LHLH, the two different interactions LH and LL are possible, and again, we find that types react according to theoretical predictions: expenditures are lower when a low type competes against a high-cost opponent, as opposed to competing against a low-cost opponent (LL: 86.14 vs. LH 73.65, $p = 0.02$).

Table 5
Random-effect panel regression of stage-one expenditures.

	(1)	(2)	(3)	(4)	(5)	(6)
	Low-cost types			High-cost types		
	LLLL and LLHH	LLLL and LHLH	LLHH and LHLH	HHHH and LLHH	HHHH and LHLH	LLHH and LHLH
Dep. variable:	stage-one expenditures			stage-one expenditures		
LLHH	11.89 (0.03) [0.03]			-5.12 (0.22) [0.46]		
LHLH		-7.57 (0.03) [0.05]	-19.46 (0.01) [0.01]		-4.08 (0.39) [0.46]	1.04 (0.84) [0.75]
Observations	4500	4500	3000	4500	4500	3000

LLHH = 1 if treatment is LLHH and zero otherwise; LHLH = 1 if treatment is LHLH and zero otherwise. All specifications contain a full set of round fixed effects. We report p-values based on standard errors clustered at the individual level in parentheses; p-values from non-parametric Mann-Whitney-U tests (N = 5 in each treatment) in squared brackets.

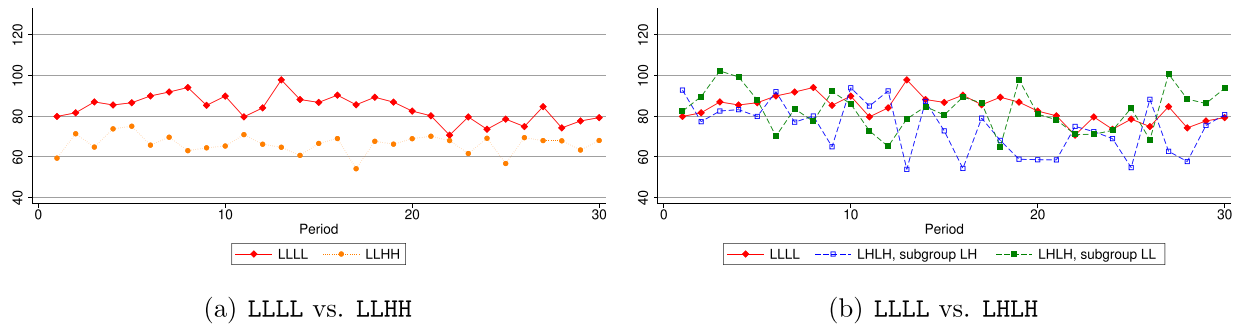


Fig. 3. Stage-Two Expenditures (Low-Cost Types).

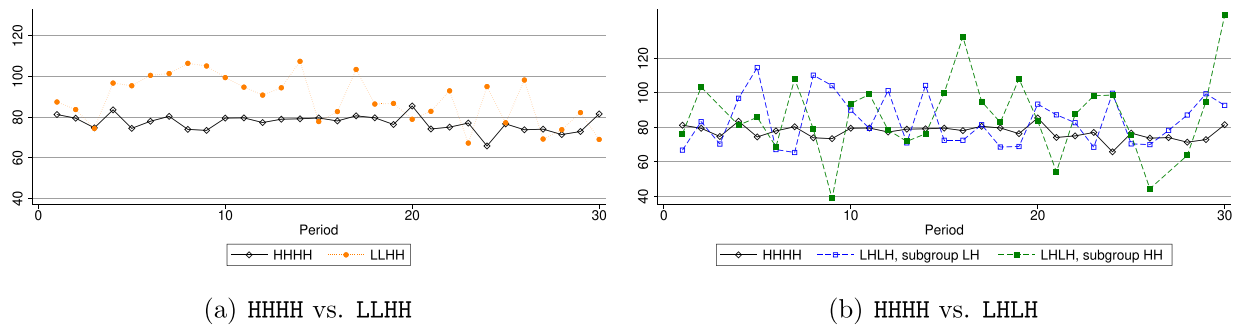


Fig. 4. Stage-Two Expenditures (High-Cost Types).

two-sample MWU-test $p = 0.12$), contrary to the prediction.¹⁴ Panel (a) of Fig. 4 shows that expenditures are not always higher — still the relationship between expenditures holds for most rounds.

Turning to the LH interaction of treatment LHLH we again see weakly higher (and not strictly lower) stage-two expenditures by high-cost agents in the LH interaction of treatment LHLH than in the HH interaction of treatment HHHH (76.61 vs. 76.32, $p = 0.77$, see column (5) in Table 6, and a two sample MWU-test, $p = 0.75$).

The comparison between the HH interaction of treatment LHLH and the HH interactions of treatment HHHH delivers an unexpected result: Expenditures are predicted to be the same, but high-cost agents spend considerably more in LHLH (87.94 vs. 76.32, $p = 0.11$, see column (6) in Table 6, and a two sample MWU-test, $p = 0.05$).

Result 5. Expenditures of high-cost agents in stage two of heterogeneous settings: *The stage-two expenditures of high-cost agents violate all parts of prediction H_5 : They spend more in the heterogeneous interaction in treatment LLHH than in the homogeneous interaction in treatment HHHH (although they are predicted to spend less), they spend the same amount in the heterogeneous interaction of treatment LHLH as in the homogeneous interaction in treatment HHHH (although they are predicted to spend less), and they spend more in the homogeneous interactions of treatment LHLH than in the homogeneous interaction in treatment HHHH (although they are predicted to spend the same amount).*

5.5. Total expenditures

Having discussed the expenditures in both stages separately, we now turn to the discussion of total expenditures. From Result 1 we would expect that total expenditures in the homogeneous treatments are independent of the cost parameter. This is indeed the case: The random-effect panel regression of the expenditures in column (1) of Table 7 indicates that total expenditures over both stages are not

¹⁴ Table A.2 in the Online Appendix shows that the results are similar when controls are included.

Table 6
Random-Effect Panel Regression of Stage-Two Expenditures.

	(1)	(2)	(3)	(4)	(5)	(6)
	Low-cost Types			High-cost Types		
	LL LLLL and LH LLHH	LL LLLL and LH LHLH	LL LLLL and LH LHLH	HH HHHH and LH LLHH	HH HHHH and LH LHLH	HH HHHH and HH LHLH
Dep. variable:	stage-two expenditures			stage-two expenditures		
LLLL	19.56 (0.00) [0.02]	9.39 (0.14) [0.34]	-1.85 (0.79) [0.92]			
HHHH				-9.86 (0.17) [0.12]	-2.62 (0.68) [0.75]	-11.50 (0.11) [0.05]
Observations	2250	1851	2160	2250	1851	1638

LLLL = 1 if treatment is LLLL and zero otherwise; HHHH = 1 if treatment is HHHH and zero otherwise. All specifications contain a full set of round fixed effects. We report p-values based on standard errors clustered at the individual level in parentheses; p-values from non-parametric Mann-Whitney-U tests (N = 5 in each treatment) in squared brackets.

Table 7
Random-Effect Panel Regression of Total Expenditures.

	(1)	(2)	(3)	(4)	(5)
	LLLL and HHHH	HOM and LLHH	HOM and LHLH	LLHH and LHLH	HOM and HET
Dep. variable:	expenditures over both stages				
LLLL	18.21 (0.16) [0.35]				
LLHH		7.51 (0.44) [0.54]		28.77 (0.08) [0.35]	
LHLH			-21.26 (0.18) [0.33]		
HET					-6.87 (0.52) [0.82]
Observations	1500	2250	2250	1500	3000

LLLL = 1 if treatment is LLLL and zero otherwise. LLHH = 1 if treatment is LLHH and zero otherwise; LHLH = 1 if treatment is LHLH and zero otherwise; HOM includes treatments LLLL and HHHH; HET includes treatments LHLH and LLHH, and is 1 if treatment is either LHLH or LLHH and zero otherwise. All specifications contain a full set of round fixed effects. P-values based on standard errors clustered at the group level are reported in parentheses; p-values from non-parametric Mann-Whitney-U tests (N = 5 in each treatment) in squared brackets.

significantly different across the homogeneous treatments ($p = 0.16$), and the result is confirmed by a two sample MWU-test, $p = 0.35$ (Table 7). This leads us to the following result:

Result 6. Total expenditures in homogeneous settings: *In line with prediction H_6 total expenditures are not significantly different between LLLL and HHHH.*

Since total expenditures do not differ significantly across the two homogeneous treatments we pool them for the comparisons with the heterogeneous treatments to HOM. According to prediction H_7 total expenditures should be lower in the heterogeneous treatments LLHH and LHLH than in HOM. Our data does not support this prediction: Total expenditures are not significantly lower in treatment LLHH than in HOM (LLHH: 303.54 vs. HOM: 296.03, $p = 0.44$, see column (2) in Table 7, and a two sample MWU-test, $p = 0.54$) and they are also not significantly lower in treatment LHLH than in HOM (LHLH: 274.76 vs. HOM: 296.03, $p = 0.18$, see column (3) in Table 7, and a two sample MWU-test, $p = 0.33$). These results do not change if we use only the data from LLLL or from HHHH (instead of using the pooled data from HOM) in the comparisons (see Table A.9 in the Online Appendix). The result

that heterogeneity does not reduce total expenditures also continues to hold if we merge the data of the two heterogeneous treatments LLHH and LHLH to HET and compare it to HOM (see column (5) in Table 7, $p = 0.52$, and a two sample MWU-test, $p = 0.82$). Comparing the two heterogeneous treatments we see that total expenditures are higher in treatment LLHH as predicted in part (iii) of H_7 , (LHLH: 274.76 vs. LLHH: 303.54, $p = 0.08$, see column (4) in Table 7, and a two sample MWU-test, $p = 0.35$).

Result 7. The effects of heterogeneity on total expenditures: *The comparisons of total expenditures violate all parts of prediction H_7 : Total expenditures are not significantly lower in the heterogeneous treatment LHLH as compared to the homogeneous benchmark HOM, they are also not significantly lower in the heterogeneous treatment LLHH as compared to the homogeneous benchmark HOM, and they are not significantly lower in the heterogeneous treatment LHLH as compared to the heterogeneous treatment LLHH.*

5.6. Discussion

The results for the two homogeneous treatments are consistent with the theoretical predictions. They also confirm existing experimental evidence that expenditures in homogeneous contests do not depend on the size of the cost parameter (see, e.g. Dechenaux et al., 2015). However, introducing heterogeneity does not have the predicted effect of reducing total expenditures. In particular, the high-cost agents do not decrease expenditures in heterogeneous settings compared to a situation where all agents have the same costs as implied by the theoretical predictions. Also, the expenditures of high-cost agents seem to be history-dependent: High-cost agents ('underdogs') who have won against a strong competitor ('favorite') in the past expend more in homogeneous interactions than high-cost agents who have won against a weak competitor in the past. This finding is distinct from round effects (which are accounted for in the empirical analysis). Overall, the deviations of high cost agents from the theoretical benchmark lead to the result that total expenditures are not lower in heterogeneous dynamic elimination contests than in their homogeneous counterparts.

Although the existing literature on heterogeneous contests is mixed and direct comparisons are hampered by the implementation of different tournament structures and sources of heterogeneity (see Dechenaux et al., 2015, for details), our finding that mainly high-cost agents do not behave as predicted in heterogeneous contests resembles earlier findings. For instance, Schotter and Weigelt (1992) found in an uneven rank-order tournament that only the disadvantaged agent exerts more effort. Using professional sports data from the Handball Bundesliga, (Berger & Nieken, 2016) analyze the impact of heterogeneous teams on the provision of investment. They show that competition is less intense between heterogeneous contestants and that this is mainly due to the fact that the favorites reduce investments whereas underdogs do not adjust their behavior. Similar results are reported by Bach et al. (2009) who use data of Olympic rowing competitions, and again find that only favorites and not underdogs react to heterogeneity.

A potential explanation for these findings is provided by a psychological aspect that is related to the utility associated with succeeding against a strong competitor. For instance, Berger and Nieken (2016) explain their results by "social or psychological costs the inferior contestant faces when not trying hard enough against an ex-ante dominant rival" (p. 22), which may be especially relevant in sports competition. While this can explain several of the deviations of high-cost agents from the theoretical benchmark in our experiments, it does not fully explain why in homogeneous stage-two interactions between high-cost agents the expenditures are history-dependent – with underdogs who have won against a favorite in stage one of the heterogeneous contest expending significantly more in the current interaction against another underdog than underdogs who have won against another underdog in stage one of the homogeneous contest. Similarly, Harbring and Lünser

(2008) point out that in a tournament with heterogeneous competitors, “an underdog strains himself all the more when competing against a more capable player while the favorite might slack off” (p. 374). Our data provides clear support for the first part of this statement: In both stages of our contests, we find that high-cost agents who face a strong competitor either do the same or even increase expenditures in comparison to the homogeneous benchmark, which is in sharp contrast to the theoretical prediction of lower expenditures. The second part of the statement (that the favorite might slack off in heterogeneous interactions) is also in line with our data — but at the same time, it is also in line with the theoretical prediction. Also in line with theory, we find that ‘favorites’ increase their expenditures in the light of a higher continuation value when they know that they will face a weak opponent in the second stage. By contrast, ‘underdogs’ barely reduce their expenditures in the light of a lower continuation value when they know that they will face a strong opponent in the second stage.

A closer inspection reveals that non-monetary payoffs related to emotional responses might indeed provide a rationalization for our findings. Several approaches along these lines exist in the literature. Kräkel (2008) addresses the role of emotions in heterogeneous contests from a theoretical perspective. Specifically, he considers the possibility that if an underdog wins against a favorite in a heterogeneous contest, the underdog will feel pride or joy (positive emotions) whereas the favorite one will be disappointed or feel shame (negative emotions). A related argument is put forward by Amaldoss and Rapoport (2009). Along similar lines, in previous work on static contests with heterogeneous contestants, Chen et al. (2011) find effort overprovision of favorites and underdogs and a stronger reaction of favorites to variation in the contest prize. To rationalize their findings, they argue that in addition to the monetary outcomes from winning or losing, agents in a contest may also care about social comparisons, i.e., “how their outcomes of winning or losing relative to others may be perceived by themselves and other contestants”. Their structural estimates reveal a greater importance of favorites experiencing pain from losing against an underdog than the joy of underdogs winning against a favorite. However, when testing an extension to emotions, they find evidence for greater overprovision among underdogs, which resembles our findings. Given that our results are from a setting with multiple stages, our findings complement theirs.

Following a similar approach to incorporate such social comparisons as in these contributions, one may allow the ‘joy of winning’ and the ‘fear of losing’ to depend on the type of the opponent, for instance by considering the possibility that underdogs derive additional utility from beating a favorite and that favorites experience a special disutility from losing against an underdog. These emotion-based explanations are consistent with the higher expenditures by underdogs in heterogeneous interactions and could potentially explain several of the deviations of high-cost agents from the theoretical benchmark in our framework. However, in the data we do not find much evidence for an expenditure-increasing effect in heterogeneous interactions among low-cost contestants. This suggests that the consideration of ‘joy of winning’ of underdogs might be more relevant to account for the empirical findings than the consideration of ‘fear of losing’. In fact, the ‘joy of winning’ extension is consistent with the three key findings, that favorites have lower expenditures in the heterogeneous than in the homogeneous interaction, that underdogs have higher expenditures in the heterogeneous than in the homogeneous interaction, and that expenditures of the underdog exceed expenditures of the favorite.¹⁵ Moreover, our evidence suggests that past emotions might influence current behavior in order to explain the result that underdogs who have won against a favorite in the past expend more in a homogeneous interaction than underdogs who have won against a weak competitor in the past. In addition, having prevailed during the previous round of the

experiment influences expenditures in the current round, but in slightly different ways for favorites and underdogs (see Tables A.1 and A.2 in the Online Appendix), even though this was in an interaction against a different opponent. This indicates that considering history and learning might be valuable directions to extending emotion-based models.

Taken together, most of the deviations from the theoretical benchmark in our data are consistent with an explanation related to positive emotions (an underdog who wins against a favorite feels pride), but not with its negative counterpart (a favorite who loses against an underdog feels shame). The results are thereby in line with the type-specific joy-of-winning hypothesis along the lines of Kräkel (2008) or (Chen et al., 2011) (an underdog derives additional utility from beating a favorite), but not with its negative counterpart (a favorite derives an additional dis-utility from losing against an underdog). Both approaches (at least in their original formulation) fail to explain why underdogs who have won against a favorite in the past expend more in a homogeneous interaction than underdogs who have won against a weak competitor in the past, as suggested by some of our findings (see Online Appendix Tables A.1 and A.2). Overall, the empirical findings suggest that the different strategic incentives in stage one and two together with the psychological factors related to an emotional response can account for the observed behavior of heterogeneous agents in our setting.

6. Concluding remarks

This paper has analyzed how heterogeneity in contestants’ investment costs affects the competition intensity in a dynamic elimination contest. Our theoretical model predicts that the level of investment costs has no effect on the expenditures in a two-stage pairwise elimination contest with homogeneous participants, whereas cost heterogeneity between competing contestants is expected to reduce aggregate expenditures. In particular, heterogeneity in the current stage is predicted to reduce expenditures of both agents, no matter whether the interaction is in stage one or stage two. In addition, theory predicts that in stage one of our two-stage contest not only the type of the direct opponent matters but also the continuation value, which is determined by the type of the opponent in the second stage.

Our experimental results are qualitatively well in line with theoretical predictions for homogeneous contests. For heterogeneous contests, we find deviations from the theoretical predictions. Responsible for the deviations from the theoretical benchmark are mainly the high-cost agents: While theory predicts that they reduce their expenditures in heterogeneous contests, they barely do so and often even increase their expenditures in our experiments. As a result, total expenditures are not lower in heterogeneous than in homogeneous two-stage elimination contests.

While our analysis has restricted attention on the four-player case with only two distinct levels of player strength – the minimal setting to study the implications of heterogeneity – we view the results as a proof of concept for a more systematic deviation in settings with more degrees of heterogeneity and more players. In the end, documenting deviations from the theoretical predictions in the most simple setup in which the question of how heterogeneity impacts contestants’ behavior can be addressed is a prerequisite for more detailed investigations. Moreover, there is little reason to expect that the behavioral deviations may change in more complex setups, which might entail greater difficulties for players to make optimal decisions.

Our results point to interesting directions for future research. In particular, behavioral factors such as type-specific positive emotions or type-specific ‘joy of winning’ constitute possible explanations. Indeed, positive emotions and type-specific ‘joy-of-winning’ of an underdog who prevails against a favorite are consistent with the observed responses of high-cost agents to heterogeneity. Different from the predictions of existing models of emotions, our evidence points at history-dependence in terms of an influence of stage-one on stage-two expenditures of high-cost agents. Moreover, we find evidence for

¹⁵ See Appendix C for details.

expenditures and success in previous rounds of the experiment having an influence on expenditures in the current round. This suggests that also past emotions might influence current behavior — an aspect that has not been explored systematically in the existing literature. Another relevant finding for models of emotions in tournaments is the asymmetry in behavior, as we find no evidence for favorites who lose against an underdog increasing their expenditures in heterogeneous interactions compared to homogeneous ones.

Data availability

Data will be made available on request.

Appendix A. Derivation of winning probability of a Low-Cost contestant

Combining the first order conditions for Eq. (3) we obtain two expressions that define a relation between equilibrium investment choices of contestants *within* each interaction, namely

$$\frac{x_{li}}{x_{lj}} = \frac{c_H}{c_L} \frac{CV_i}{CV_j} \quad \text{and} \quad \frac{x_{lk}}{x_{li}} = \frac{c_H}{c_L} \frac{CV_k}{CV_i}. \tag{11}$$

These expressions show that each stage-one interaction is a tournament between agents with different costs and endogenously different valuations of winning. While the costs of investment differ by construction, the difference of the value for winning is a result of the tournament structure: Reaching stage two is more valuable for low-cost than for high-cost contestants.

The continuation values are endogenously determined by the probabilities of entering the different actions in stage two (which are determined by the investments in the other pairwise stage-one interaction, $\frac{x_{lk}}{x_{lk}+x_{li}}$) and the respective equilibrium payoffs: Whereas continuation values in the homogeneous settings and in the setting LLHH are determined by the payoff of the unique possible interaction in stage two, there are 3 different possible interactions in stage two of setting LHLH – namely LH, LL or HH. Thus, continuation values are expected values in this case, and read as follows:

$$CV_i = \frac{x_{lk}}{x_{lk} + x_{li}} \cdot \Pi_{2L}^*(LL) + (1 - \frac{x_{lk}}{x_{lk} + x_{li}}) \cdot \Pi_{2L}^*(LH); \tag{12}$$

$$CV_j = \frac{x_{lk}}{x_{lk} + x_{li}} \cdot \Pi_{2H}^*(LH) + (1 - \frac{x_{lk}}{x_{lk} + x_{li}}) \cdot \Pi_{2H}^*(HH). \tag{13}$$

Similarly, the continuation values in the other stage-two interaction between contestant *k* and *l* depend on the behavior of agents *i* and *j*.

As mentioned previously, any tournament with two heterogeneous participants has a unique, interior equilibrium for the chosen contest success function (Cornes & Hartley, 2005; Nti, 1999). Consequently, each of the two pairwise stage-one interactions has a unique equilibrium for each pair of continuation values. What remains to be shown is that the two expressions in (11) can be satisfied *jointly*. Inserting (13) into (11) and simplifying gives

$$\frac{x_{li}}{x_{lj}} = \frac{c_H}{c_L} \frac{(c_L + c_H)^2 \frac{x_{kl}}{x_{li}} + 4c_H^2}{4c_L^2 \frac{x_{kl}}{x_{li}} + (c_L + c_H)^2} \quad \text{and} \quad \frac{x_{kl}}{x_{li}} = \frac{c_H}{c_L} \frac{(c_L + c_H)^2 \frac{x_{il}}{x_{li}} + 4c_H^2}{4c_L^2 \frac{x_{il}}{x_{li}} + (c_L + c_H)^2}. \tag{14}$$

System (14) consists of two equations in the two unknowns $\frac{x_{li}^*}{x_{lj}^*}$ and $\frac{x_{lk}^*}{x_{li}^*}$, respectively. Note that the two equations are symmetric, since the two contestants in each of the two stage-one interactions face identical optimization problems. This implies that $x_{iL}^* \equiv x_{li}^* = x_{lk}^*$ and $x_{iH}^* \equiv x_{lj}^* = x_{li}^*$, in the symmetric equilibrium.¹⁶ Combining these conditions

¹⁶ The symmetric equilibrium exists for any degree of heterogeneity and is unique. Intuitively, one must show that the graphs of the two relations in (14)

with (14) gives:

$$\begin{aligned} \frac{x_{iL}^*}{x_{iH}^*} &= \frac{c_H}{c_L} \frac{(c_L + c_H)^2 \frac{x_{iL}^*}{x_{iH}^*} + 4c_H^2}{4c_L^2 \frac{x_{iL}^*}{x_{iH}^*} + (c_L + c_H)^2} \\ \Leftrightarrow 0 &= 4c_L^2 \left[\frac{x_{iL}^*}{x_{iH}^*} \right]^2 + \left(1 - \frac{c_H}{c_L} \right) (c_L + c_H)^2 \left[\frac{x_{iL}^*}{x_{iH}^*} \right] - 4 \frac{c_H^3}{c_L} \\ \Leftrightarrow \frac{x_{iL}^*}{x_{iH}^*} &= F^*(c_L, c_H), \text{ where} \\ F^*(c_L, c_H) &= \frac{(c_H - c_L)(c_L + c_H)^2 + \sqrt{64c_H^3c_L^3 + (c_L - c_H)^2(c_L + c_H)^4}}{8c_L^3}. \end{aligned} \tag{15}$$

$F^*(c_L, c_H)$ is the ratio of stage-one investments of the two types of contestants. It is directly proportional to heterogeneity in costs and continuation values, as Eq. (11) shows. Therefore, $F^*(c_L, c_H)$ can be interpreted as a measure for both the exogenous heterogeneity in investment costs between low-cost and high-cost contestants and the endogenous heterogeneity between types that is due to different continuation values in stage one. Using this expression we can redefine the probability that a low-cost contestant wins as

$$\begin{aligned} q_L^* &= \frac{x_{iL}^*}{x_{iL}^* + x_{iH}^*} \\ &= \frac{F^*}{1 + F^*} \\ &= \frac{(c_H - c_L)(c_L + c_H)^2 + \sqrt{64c_H^3c_L^3 + (c_L - c_H)^2(c_L + c_H)^4}}{(c_H - c_L)(c_L + c_H)^2 + \sqrt{64c_H^3c_L^3 + (c_L - c_H)^2(c_L + c_H)^4} + 8c_L^3}. \end{aligned}$$

Appendix B. Proof of Proposition 2

B.1. Prerequisites

For the proof of Proposition 2, we make use of the following three lemmata. In the derivation, we assume without loss of generality that $c_H \geq c_L = 1$.

Lemma 1. Define $f(c_H) = \frac{5c_H^3 + 2c_H^2 + c_H}{c_H^2 + 2c_H + 5}$. Then, the relation $F^*(1, c_H) > f(c_H)$ holds for all $c_H > 1$. Furthermore, for $c_H = 1$ it holds that $F^*(1, c_H) = f(c_H)$.

Proof. From Eq. (11) we know that $\frac{x_{li}}{x_{lj}} = \frac{c_H}{c_L} \frac{CV_i(x_{lk}, x_{li})}{CV_j(x_{lk}, x_{li})}$. Further $\frac{x_{li}^*}{x_{lj}^*} = F^*(c_L, c_H)$. Consequently, using the assumption that $c_H \geq c_L = 1$, it must hold that

$$F^*(1, c_H) = \frac{CV_i(x_{lk}, x_{li})}{CV_j(x_{lk}, x_{li})} = \frac{4c_H^3 + c_H(1 + c_H)^2 \frac{x_{lk}}{x_{li}}}{(1 + c_H)^2 + 4 \frac{x_{lk}}{x_{li}}}.$$

Note that for $c_H > 1$, we have

$$\frac{\partial F^*(1, c_H)}{\partial \frac{x_{lk}}{x_{li}}} = \frac{(1 + c_H)^4 - 16c_H^2}{[(1 + c_H)^2 + 4 \frac{x_{lk}}{x_{li}}]^2} > 0.$$

Further, recall that player 1 has both higher cost ($c_H > 1$) and a lower continuation value ($CV_k > CV_j$), such that $x_{lk} > x_{li}$ does hold. Therefore, assuming $x_{lk} = x_{li}$ underestimates $F^*(1, c_H)$. Since

$$f(c_H) = \frac{5c_H^3 + 2c_H^2 + c_H}{c_H^2 + 2c_H + 5}$$

have a unique intersection in the domain defined by $\frac{x_{li}^*}{x_{lj}^*} \in [0, 1]$ and $\frac{x_{lk}^*}{x_{li}^*} \in [0, 1]$. It suffices to consider this domain, since the assumption of lower investment costs and the resulting higher value of winning of contestants with low costs imply $x_{li}^* \geq x_{lj}^*$ and $x_{lk}^* \geq x_{li}^*$, respectively. This follows from (11). Details available upon request from the corresponding author.

is the expression we derive from $F^*(1, c_H)$ under this assumption, we have proven $F^*(1, c_H) > f(c_H)$. If we assume $c_H = 1$, all players are perfectly symmetric, such that $x_{1k} = x_{1l}$ does hold. Consequently, the relation $F^*(1, c_H) = f(c_H)$ does hold for $c_H = 1$. \square

Lemma 2. Define $f^l(c_H) = 2c_H - 1$. Then, the relation $F^*(1, c_H) > f^l(c_H)$ holds for all $c_H > 1$. Furthermore, for $c_H = 1$, it holds that $f(c_H) = f^l(c_H)$.

Proof. We start with the relation that we want to prove, namely:

$$\begin{aligned} f(c_H) &> f^l(c_H) \\ \Leftrightarrow 5c_H^3 + 2c_H^2 + c_H &> (2c_H - 1)(c_H^2 + 2c_H + 5) \\ \Leftrightarrow 3c_H^3 - c_H^2 - 7c_H + 5 &> 0 \end{aligned}$$

We now have to prove that $\phi(c_H) \equiv 3c_H^3 - c_H^2 - 7c_H + 5 > 0$ does always hold for $c_H > 1$. To see this, note that $\phi(\cdot)$ is a cubic function that has a local minimum at $c_H = 1$, and a local maximum at $c_H = -7/9$. Furthermore, $\phi(1) = 0$, which implies that $\phi(c_H) > 0$ for all $c_H > 1$. \square

Lemma 3. Assume without loss of generality that $c_H \geq c_L = 1$ and define $f_h(c_H) = \frac{c_H^3 + 2c_H^2 + c_H}{4}$. Then, the relation $F^*(1, c_H) < f^h(c_H)$ does hold for all $c_H > 1$. Furthermore, for $c_H = 1$, it holds that $F^*(1, c_H) = f^h(c_H)$.

Proof. From Eq. (11), we know that $\frac{x_{1l}}{x_{1j}} = \frac{c_H CV_j(x_{1k}, x_{1l})}{c_L CV_j(x_{1k}, x_{1l})}$. Further, Eq. (15) tells us that $\frac{x_{1l}}{x_{1j}} = F^*(c_L, c_H)$. Consequently, using the assumption that $c_H \geq c_L = 1$, it must hold that

$$F^*(1, c_H) = c_H \frac{CV_i(x_{1k}, x_{1l})}{CV_j(x_{1k}, x_{1l})} = \frac{4c_H^3 \times \frac{x_{1l}}{x_{1k}} + c_H(1 + c_H)^2}{(1 + c_H)^2 \times \frac{x_{1l}}{x_{1k}} + 4}$$

Note that for $c_H > 1$, we have

$$\frac{\partial F^*(1, c_H)}{\partial \frac{x_{1l}}{x_{1k}}} = -\frac{(c_H - 1)^2 c_H (c_H^2 + 6c_H + 1)}{[(1 + c_H)^2 \times \frac{x_{1l}}{x_{1k}} + 4]^2} < 0.$$

Further, recall that player 1 will never drop out in a pairwise competition for any finite degree of heterogeneity in terms of costs and continuation value, such that $x_{1l} > 0$ does hold. Therefore, assuming $x_{1l} = 0$ (which implies $\frac{x_{1l}}{x_{1k}} = 0$) overestimates $F^*(1, c_H)$, since this expression is decreasing in $\frac{x_{1l}}{x_{1k}}$. Since

$$f^h(c_H) = \frac{c_H^3 + 2c_H^2 + c_H}{4}$$

is the expression we derive from $F^*(1, c_H)$ under this assumption, we have proven $F^*(1, c_H) < f^h(c_H)$. If we assume $c_H = 1$, all players are perfectly symmetric, such that $x_{1l} = x_{1k}$ does hold. When inserting this relation in $F^*(1, c_H)$, we see that the relation $F^*(1, c_H) = f^h(c_H)$ does hold for $c_H = 1$. \square

B.2. Proof of Proposition 2

We have to show that the following relations hold

- (a) $TE(\text{HOM}) > TE(\text{LLHH})$
- (b) $TE(\text{HOM}) > TE(\text{LHLH})$
- (c) $TE(\text{LHLH}) > TE(\text{LLHH})$

Consider first part (a). Here the stage-two interaction in the heterogeneous setting is for sure between a low-cost and a high-cost type, thus the total expenditure in equilibrium reads as follows:

$$TE(\text{LLHH}) = [(2x_{1L}^* c_L + 2x_{1H}^* c_H) + (x_{2L}^* c_L + x_{2H}^* c_H)] \quad (16)$$

Without loss of generality we can set $c_L = 1$. By inserting equilibrium investments from 6 and 2.1 and simplifying we get

$$TE(\text{LLHH}) = \frac{(4c_H + c_H^2 + 1)}{2(1 + c_H)^2} \cdot R$$

This is a quadratic expression in c_H with its maximum at $c_H = 1$ which would represent the homogeneous setting where $c_H = c_L = 1$. Thus, the sum of the expenditures are always smaller whenever $c_H > c_L = 1$.

For part (b) we have to show that the expenditure of the heterogeneous setting LHLH is again smaller than in the homogeneous settings. For part (c) we have to show that the relation $TE(\text{LLHH}) > TE(\text{LHLH})$ holds.

We will proceed in two steps. First, we derive a necessary and sufficient condition in terms of the function $F^*(1, c_H)$ for the relation $TE(\text{LLHH}) > TE(\text{LHLH})$ to hold. Second, we prove that the equilibrium function $F^*(1, c_H)$, which was derived in (15) indeed satisfies this condition. We start with the relation which we want to prove:

$$TE(\text{LLHH}) > TE(\text{LHLH})$$

where $TE(\text{LLHH})$ is defined as in (16) and

$$\begin{aligned} TE(\text{LHLH}) &= \frac{2(c_L + c_H)^2 [F^*(c_L, c_H)^2 + F^*(c_L, c_H)] + 8c_H^2 F^*(c_L, c_H) + 8c_L^2 F^*(c_L, c_H)^2}{4(c_L + c_H)^2 [1 + F^*(c_L, c_H)]^3} R \\ &+ [2(q_L^2 x_{2L}(\text{LL})c_L + (1 - q_L)^2 x_{2H}(\text{HH})c_H \\ &+ q_L(1 - q_L)(x_{2L}(\text{LH})c_L + x_{2H}(\text{LH})c_H))] \end{aligned}$$

Inserting equilibrium expressions and simplifying yields

$$\begin{aligned} &\frac{c_H^2 + 4c_H + 1}{2(c_H + 1)^2} \\ &> \frac{(4c_H^3 + c_H^2 + 2c_H) + (c_H + 1)^2 F^*(1, c_H)^3 + (2c_H^2 + 12c_H + 2) F^*(1, c_H)^2}{2(c_H + 1)^2 (1 + F^*(1, c_H))^3} \\ &+ \frac{(c_H^3 + 7c_H^2 + 11c_H) F^*(1, c_H) + F^*(1, c_H) + 1}{2(c_H + 1)^2 (1 + F^*(1, c_H))^3} \end{aligned}$$

Multiplying both sides by $2(c_H + 1)^2 (1 + F^*(1, c_H))^3$ and rearranging gives

$$\begin{aligned} &F^*(1, c_H)^3 \cdot 2c_H + F^*(1, c_H)^2 \cdot (c_H^2 + 1) \\ &+ F^*(1, c_H) \cdot (-c_H^3 - 4c_H^2 + c_H + 2) + 2c_H - 4c_H^3 > 0. \end{aligned}$$

Solving for $F^*(1, c_H)$ gives us following roots:

$$\begin{aligned} r_1 &= \frac{-1 + c_H^2 - \sqrt{1 - 16c_H - 2c_H^2 + 32c_H^3 + c_H^4}}{4c_H} \\ r_2 &= \frac{-1 + c_H^2 + \sqrt{1 - 16c_H - 2c_H^2 + 32c_H^3 + c_H^4}}{4c_H} \\ r_3 &= -c_H \end{aligned}$$

We do only have to consider r_2 , since r_1 and r_3 are below 0 for some values of c_H , while $F^*(1, c_H) \geq 1$ for all $c_H \geq 1$.¹⁷ Thus we have to show that

$$F^*(1, c_H) > r_2 \equiv \frac{-1 + c_H^2 + \sqrt{1 - 16c_H - 2c_H^2 + 32c_H^3 + c_H^4}}{4c_H}, \quad (17)$$

for all $c_H > 1$. From Lemmata 1 and 2 we know that $F^*(1, c_H) > f^l(c_H)$. Consequently, a sufficient condition for (17) is given by $f^l(c_H) > r_2$. Using the expression $f^l(c_H)$ and rearranging gives $7c_H^2 - 4c_H + 1 > \sqrt{1 - 16c_H - 2c_H^2 + 32c_H^3 + c_H^4}$. Squaring both sides leaves us with¹⁸

$$2c_H(25c_H^3 - 12c_H^2 + 14c_H - 12) > 0$$

This relation is always satisfied if $c_H > 1$, which completes the proof.

We proved the relation $TE(\text{LLHH}) > TE(\text{LHLH})$ and from part (a) of the proof we know $TE(\text{hom}) > TE(\text{LLHH})$ thus $TE(\text{hom}) > TE(\text{LHLH})$ is also true.

¹⁷ It follows from Lemma 1 that $\frac{\partial F^*(1, c_H)}{\partial c_H} > 0$, and $F^*(1, 1) = 1$ holds. Therefore, $F^*(1, c_H) \geq 1$ for all $c_H \geq 1$.

¹⁸ Note that squaring is without loss of generality here, since we are only interested in solutions for $c_H > 1$.

Appendix C. Extension: Emotions

In the following, we provide a brief discussion of the implications of extending the model to emotions, in terms of “anger” (negative emotions perceived by a low-cost player, the favorite, in case of losing against a high-cost player, the underdog) or “joy” (positive emotions perceived by the underdog in case of winning against a favorite). We restrict attention to interactions on the second stage. Our exposition is along the lines of Kräkel (2008).

(1) Homogeneous and heterogeneous without emotions

If perceived and monetary prizes are identical, then the formal maximization problem of player i in stage 2 reads

$$\max \Pi_{2i}(x_{2i}, x_{2j}) = \frac{x_{2i}}{x_{2i} + x_{2j}} R - c_i x_{2i}$$

$$\frac{\partial \Pi_{2i}}{\partial x_{2i}} = \frac{x_{2j}}{(x_{2i} + x_{2j})^2} R - c_i = 0$$

yields

$$x_{2i}^* = \frac{c_j}{(c_i + c_j)^2} R; \quad x_{2j}^* = \frac{c_i}{(c_i + c_j)^2} R.$$

Following (Kräkel, 2008) we assume below that emotions play only a role in asymmetric interactions.

In asymmetric interactions, let

- f denote the favourite,
- u denote the underdog.

With this notation, the equilibrium efforts and expenditures in asymmetric interactions without emotions read

$$x_{2f}^* = \frac{c_u}{(c_f + c_u)^2} R; \quad x_{2u}^* = \frac{c_f}{(c_f + c_u)^2} R.$$

$$E_{2f}^* = \frac{c_f c_u}{(c_f + c_u)^2} R; \quad E_{2u}^* = \frac{c_u c_f}{(c_u + c_f)^2} R$$

(2) Anger only

Suppose the favorite feels anger $A \geq 0$ when losing against the underdog in an asymmetric interaction (while the underdog does not feel any emotions). Then

- the favorite maximizes

$$\Pi_{2f}(x_{2f}, x_{2u}) = \frac{x_{2f}}{x_{2f} + x_{2u}} R - \frac{x_{2u}}{x_{2f} + x_{2u}} A - c_f x_{2f}$$

$$\frac{\partial \Pi_{2f}}{\partial x_{2f}} = \frac{x_{2u}}{(x_{2f} + x_{2u})^2} (R + A) - c_f = 0$$

- the underdog maximizes

$$\max \Pi_{2u}(x_{2u}, x_{2f}) = \frac{x_{2u}}{x_{2u} + x_{2f}} R - c_u x_{2u}$$

$$\frac{\partial \Pi_{2u}}{\partial x_{2u}} = \frac{x_{2f}}{(x_{2u} + x_{2f})^2} R - c_u = 0$$

yields

$$x_{2f}^* = \frac{c_u R(R + A)^2}{(c_u R + c_u A + c_f R)^2}; \quad x_{2u}^* = \frac{c_f R^2(R + A)}{(c_u R + c_u A + c_f R)^2}.$$

$$E_{2f}^* = \frac{c_f c_u R(R + A)^2}{(c_u R + c_u A + c_f R)^2}; \quad E_{2u}^* = \frac{c_f c_u R^2(R + A)}{(c_u R + c_u A + c_f R)^2}$$

Both x_{2f}^* and E_{2f}^* increase in A , while x_{2u}^* and E_{2u}^* decrease in A .
 \Rightarrow with anger only, heterogeneity

- does not necessarily decrease the expenditures of the favorite (and if it decreases them, it decreases them by less than without emotions)
- necessarily decreases the expenditures of the underdog (and the negative effect of heterogeneity on expenditures is larger than without emotions)

- expenditures of the favorite exceed expenditures of the underdog

(3) Joy only

Suppose the underdog feels joy $J \geq 0$ when winning against the favorite in an asymmetric interaction (while the favorite does not feel any emotion). Then

- the favorite maximizes

$$\Pi_{2f}(x_{2f}, x_{2u}) = \frac{x_{2f}}{x_{2f} + x_{2u}} R - c_f x_{2f}$$

$$\frac{\partial \Pi_{2f}}{\partial x_{2f}} = \frac{x_{2u}}{(x_{2f} + x_{2u})^2} R - c_f = 0$$

- the underdog maximizes

$$\max \Pi_{2u}(x_{2u}, x_{2f}) = \frac{x_{2u}}{x_{2u} + x_{2f}} (R + J) - c_u x_{2u}$$

$$\frac{\partial \Pi_{2u}}{\partial x_{2u}} = \frac{x_{2f}}{(x_{2u} + x_{2f})^2} (R + J) - c_u = 0$$

yields

$$x_{2f}^* = \frac{c_u R^2(R + J)}{(c_u R + c_f R + c_f J)^2}; \quad x_{2u}^* = \frac{c_f R(R + J)^2}{(c_u R + c_f R + c_f J)^2}$$

$$E_{2f}^* = \frac{c_f c_u R^2(R + J)}{(c_u R + c_f R + c_f J)^2}; \quad E_{2u}^* = \frac{c_f c_u R(R + J)^2}{(c_u R + c_f R + c_f J)^2}$$

Both x_{2f}^* and E_{2f}^* increase in J , and also x_{2u}^* and E_{2u}^* increase in J .
 \Rightarrow with joy only, heterogeneity

- does not necessarily decrease the expenditures of the favorite (and if it decreases them, it decreases them by less than without emotions)
- does not necessarily decrease the expenditures of the underdog (and if it decreases them, it decreases them by less than without emotions)
- expenditure of the underdog exceeds expenditure of the favorite

(4) Anger and joy

Suppose the favorite feels anger $A \geq 0$ when losing against the underdog and the underdog feels joy $J \geq 0$ when winning against the favorite. Then

- the favorite maximizes

$$\Pi_{2f}(x_{2f}, x_{2u}) = \frac{x_{2f}}{x_{2f} + x_{2u}} R - \frac{x_{2u}}{x_{2f} + x_{2u}} A - c_f x_{2f}$$

$$\frac{\partial \Pi_{2f}}{\partial x_{2f}} = \frac{x_{2u}}{(x_{2f} + x_{2u})^2} (R + A) - c_f = 0$$

- the underdog maximizes

$$\max \Pi_{2u}(x_{2u}, x_{2f}) = \frac{x_{2u}}{x_{2u} + x_{2f}} (R + J) - c_u x_{2u}$$

$$\frac{\partial \Pi_{2u}}{\partial x_{2u}} = \frac{x_{2f}}{(x_{2u} + x_{2f})^2} (R + J) - c_u = 0$$

yields

$$x_{2f}^* = \frac{c_u (R + A)^2 (R + J)}{[(R + A)c_u + (R + J)c_f]^2}; \quad x_{2u}^* = \frac{c_f (R + J)^2 (R + A)}{[(R + A)c_u + (R + J)c_f]^2}$$

$$E_{2f}^* = \frac{c_u c_f (R + A)^2 (R + J)}{[(R + A)c_u + (R + J)c_f]^2}; \quad E_{2u}^* = \frac{c_u c_f (R + J)^2 (R + A)}{[(R + A)c_u + (R + J)c_f]^2}$$

For stage 2 of our experiment we find (qualitatively)

- (a) favorites have lower expenditures in the heterogeneous than in the homogeneous interaction
- (b) underdogs have higher expenditures in the heterogeneous than in the homogeneous interaction

(c) expenditures of the underdog exceed expenditures of the favorite.

Findings (b) and (c) are inconsistent with the anger only model.

Findings (a), (b), and (c) are consistent with the joy only model.

More generally, findings (a) and (b) are easier to bring in line with the anger and joy model if we set $A = 0$ (otherwise A must be very small to guarantee lower expenditures of the favorite in the heterogeneous than in the homogeneous interaction and at the same time A must be small and J must be very large to guarantee higher expenditures of the underdog in the heterogeneous than in the homogeneous interaction). Also, finding (c) is only consistent with the anger and joy model if $J > A$.

Appendix D. Supplementary data

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.socec.2023.102029>.

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