# Inflation with sterile scalar coupled to massive fermions and to gravity

Jéssica S. Martins

*Instituto de Física Teórica, Universidade Estadual Paulista*
*- Campus de São Paulo, 01140-070 São Paulo, Brazil*

Oliver F. Piattella

*Núcleo Cosmo-UFES & Departamento de Física,*
*Universidade Federal do Espírito Santo, Avenida Fernando Ferrari 514,*
*29075-910 Vitória, Espírito Santo, Brazil*

Ilya L. Shapiro

*Departamento de Física, ICE, Universidade Federal de Juiz de Fora*
*Campus Universitário - Juiz de Fora,*
*36036-330, Minas Gerais, Brazil and*
*Tomsk State Pedagogical University, 634061, Tomsk,*
*Russia and Tomsk State University, 634050, Tomsk, Russia*

Alexei A. Starobinsky

*L. D. Landau Institute for Theoretical Physics RAS, Moscow, 119334, Russia and*
*National Research University Higher School of Economics, Moscow, 101000, Russia*

(Dated: November 8, 2021)

## Abstract

In the recent paper [3] it was shown that the consistency of the quantum theory of a sterile scalar coupled to massive fermions requires the inclusion of odd-power terms in the potential of scalar self-interaction. One of the most important examples of a sterile scalar is the inflaton, that is typically a real scalar field which does not belong to representations of particle physics gauge groups, such as $SU(2)$. Here we explore the effects of the odd-power terms in the inflation potential on main observables, such as the scalar spectral index $n_s$ and the tensor-to-scalar ratio $r$, in the case in which the inflaton is strongly and non-minimally coupled to gravity. It is shown that the predicted $n_s$ deviates from the standard $-2/N$ value (corresponding to the simplest one-parametric viable inflationary models) by terms proportional to the new couplings of the odd-power terms, among which the largest one and potentially detectable is $g/N^{1/2}$, where $g$ is the coupling of the self-interaction $\varphi^3$.



Keywords: Inflaton, sterile scalar, inflation, cosmological observables

## I. INTRODUCTION

Inflaton-based models play an important role in the understanding of inflation. There is an extensive variety of inflaton models [1], however typically the inflaton is a specially designed real scalar field with the potential providing a special dynamics of the vacuum, such that the Universe can inflate in a proportion required by existing observational data. Since the inflaton is a scalar which is not a representation of a gauge group of the Standard Model, it can be called a sterile scalar. On the other hand, this scalar has to be coupled to ordinary matter in order for the reheating phase to take place at the end of the inflationary period [2].

Recently, it was shown that a sterile scalar coupled to massive fermions has to satisfy certain consistency conditions, related to quantum corrections [3]. In particular, the renormalizability of such theory can be achieved only if the inflaton potential is supplemented by three terms which have odd powers of the sterile scalar field. A relevant detail is that these odd-power terms are not necessary in other models e.g. in the Higgs inflation [4]. In this model the loop corrections are also important, as they define the value of the non-minimal parameter $\xi$ and even impose the constraints on the Higgs mass (see e.g. [5, 6] and [7–9]), but there is no need to include odd terms, since the Higgs field is not a sterile field.

The situation described above opens the following interesting possibility. Up to some extent, the inflaton-based models can be mapped to the $f(R)$-type modified theory of gravity. In general, this requires a conformal transformation (or even two of them in case of a non-minimal coupling of inflaton to gravity), but in the case of strong non-minimal coupling, this can be achieved for inflationary trajectories in the phase space even without using it. This was done for example in [10] for the case of the $\alpha$-Attractors class of inflationary models [11], and in [12] for the mixed Higgs-$R^2$ inflationary model. This feature certainly remains valid for the inflaton models with odd potential. But if the inflaton is a real field, after such a mapping we shall meet very specific additions to the function $f(R)$ that may produce observables which are different from the ones of other, most frequently used, functions. As far as odd terms in the potential are typical only for the inflaton-based models, one can use the observable consequences of these terms to learn whether inflation is caused by the inflaton, or by some form of modified gravity theory.

In the present work we shall explore this possibility. To do so, we use the following strategy. We map the inflaton potential with odd terms to a $f(R)$ gravity model without a conformal transformation, i.e. staying in the same Jordan frame. This can be done approximately in the case of strong non-minimal inflaton coupling to gravity and of sufficiently smooth behaviour of the inflaton, such as the one during slow-roll inflation. Thus, particle masses, the Hubble parameter and space-time curvature keep their original physical values during this mapping. The corresponding function $f(R)$ has additional terms due to the odd powers in the inflaton potential, and further work will concern these additional terms. Since the odd terms are assumed to be numerically small, the resulting $f(R)$ will be a sum of the usual $R^2$-term, typical of the Starobinsky model of inflation [13, 14], plus extra terms which produce the effect of our interest.

The paper is organized as follows. In Sec. II we use the perturbation theory and conformal transformation to perform the mapping of the original potential with odd terms [3] to the Einstein frame, that is the most standard way for the analysis of inflationary parameters. An alternative mapping without the use of a conformal transformation is considered in Sec. III. It leads to the $f(R)$ model having the same inflationary stage with the same predictions for primordial perturbation spectra as the original model. In Sec. IV we derive and analyze the inflationary slow-roll parameters. Finally, in Sec. V we draw our conclusions.

## II. SCALAR FIELD WITH ODD TERMS AND TRANSFORMATION TO THE EINSTEIN FRAME

Assuming that the inflaton (or other kinds of sterile scalar) $\varphi$ couples to fermions $\psi_k$ by means of a Yukawa-type interaction, i.e., $h_k\bar{\psi}_k\varphi\psi_k$, the picture is qualitatively different from the one for the fermion-Higgs interactions. The Higgs scalar belongs to the fundamental representation of the $SU(2)$ gauge group and therefore has the corresponding group index, $\Phi = \Phi_i$, with $i = 1, 2$. As a result, the divergences of the odd powers of $\Phi$ are forbidden. For a sterile scalar this is not the case and one can expect that the corresponding divergences show up, according to power counting. The explicit calculations using the heat-kernel method [3] have shown that the corresponding counterterms emerge already at the one-loop level. According to standard arguments, this means that the odd terms should be included already at the classical level, in order for the theory to be renormalizable. If we do not follow this standard procedure, the odd terms will emerge anyway, proportional to the leading logarithms in momenta or scalar field, and will be more difficult to control.

Taking into account the non-minimal interaction between the sterile scalar field and the scalar curvature, the potential of the sterile scalar reads

$$\begin{aligned} V(\varphi) \;=&\; \frac{1}{2}m^2\varphi^2 + \frac{\lambda}{4!}\varphi^4 - \frac{1}{2}\xi\varphi^2 R \\ &+ \frac{g}{3!}\varphi^3 - \tau\varphi + \tilde{g}R\varphi, \end{aligned} \tag{1}$$

where $\lambda$ is the usual dimensionless scalar self-coupling parameter and $m$ is the scalar mass. The three terms containing $\varphi$, $\varphi^3$ and $R\varphi$ are the novel elements of the model involved compared to a Higgs-like potential with $V(\varphi) = V(-\varphi)$ and standard non-minimal coupling to gravity $\propto R\varphi^2$. Their appearance follows from the renormalizabity in the flat space-time if interaction with fermions is present as was discussed in the Introduction. For the same renormalizability requirement, we do not introduce higher order odd powers of $\varphi$ and do not consider alpha-attractor models like $V = V_0 \tanh^2(a\varphi)$ for which viable inflationary models can be constructed even in the absence of non-minimal coupling to gravity $\xi = 0$. The minimal inflationary models, like the Higgs inflation and the Starobinsky inflation, have only one free parameter related to gravity ($\xi$ in the former case) which value is unambiguously fixed by the measured amplitude of the primordial power spectrum of scalar perturbations. In our model, we have three additional parameters $g$, $\tilde{g}$ and $\tau$ ($m^2$ does not contribute to the leading terms in the expressions for $n_s - 1$ and $r$) from which only one ($\tilde{g}$) is related to gravity. Thus, the model still has significant predictive power. This means that new mass scales appear in our problem. Since these mass scales are not invariant under the conformal transformation, we have to present our novel final results for them in the original Jordan frame.

We are using the notations $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$. As a consequence, $R > 0$ during inflation, and so $\xi > 0$ is the necessary condition for the non-minimal coupling. In fact, we assume strong non-minimal coupling, $\xi \gg 1$, similar to what is required for the Higgs inflationary model, since otherwise it is not possible to have primordial scalar perturbations being small in the regime of weak coupling when $|\lambda| \ll 1$, but not too small. On the top of this, we need $\lambda > 0$ for the stability of the vacuum state. Furthermore, $g$, $\tau$ and $\lambda$ are the non-minimal parameters corresponding to the odd powers of the scalar. Different from $\lambda$ and $\xi$ these parameters are dimensional, $[g] = [mass]$, $[\tilde{g}] = [mass]$ and $[\tau] = [mass^3]$.

The analysis of the renormalization group equations for $g$, $\tau$ and $\tilde{g}$ shows that the minimal possible magnitudes of these parameters are defined by the masses of heaviest fermions, i.e. the top quark, in the Standard Model. Compared to the value of the Hubble parameter, even at the end of inflation, these values are small. However, even relatively small parameters can produce measurable effects if the corresponding terms are qualitatively different. Thus, in what follows we shall try to explore such traces for the odd terms in the potential (1). Note that though $|V_1| \ll V_0$ during inflation, the contribution of $V_1$ to small observable quantities $|n_s - 1|$ and $r$ is not small compared to that from $V_0$ due to the specific symmetry of the problem (the approximate flatness of the inflaton potential in the Einstein frame, or closeness of the function $f(R)$ to $AR^2/M_P^2$ with $A$ being dimensionless and large in the approximate $f(R)$ representation of the problem in the original Jordan frame).

As a first step, let us transform the action of gravity and non-minimal scalar with potential (1),

$$S = \int d^4x\sqrt{-g}\bigg\{\frac{M_P^2}{2}R + \frac{1}{2}\,g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\bigg\}, \tag{2}$$

into the Einstein frame. We assume that the terms $V_0(\varphi) = \frac{\lambda}{4!}\varphi^4 - \frac{1}{2}\xi R\varphi^2$ are dominating and treat the rest of the potential,

$$V_1(\varphi) = \frac{1}{2}m^2\varphi^2 + \frac{g}{3!}\varphi^3 - \tau\varphi + \tilde{g}R\varphi,$$

as a small perturbation, taking only first order effects into account. Also, during the inflationary epoch, the kinetic term can be neglected, so we shall not take it into account, even as a perturbation. Though our calculations could be equally well done using the total sum $V_0 + V_1$, this would only complicate the answer, since it is known already that $V_0$ produces a very good fit to the measured value of $n_s - 1$. Thus, any additional contribution from $V_1$ should be small, that is why we consider it as a small perturbation relative to $V_0$. We shall denote the solution of the corresponding equation of motion with $V_0(\varphi)$ as $\varphi_0$, and the solution of the full equation, with $V(\varphi) = V_0(\varphi) + V_1(\varphi)$, as $\varphi_0 + \varphi_1$.

First we consider the theory with the basic potential $V_0(\varphi)$. It proves useful to perform the following change of variables in the zero-order action:

$$M_P^2 B \,=\, \xi\varphi_0^2, \tag{3}$$

where $B$ is a new scalar field. Then the reduced (without the kinetic term) form of the action (2) is

$$S_0 \,=\, \int d^4x\sqrt{-g}\bigg\{\frac{M_P^2}{2}(B+1)R - \frac{M_P^2}{4\alpha}B^2\bigg\}, \tag{4}$$

where $\alpha$ is the first of the useful new parameters

$$\alpha = \frac{3\xi^2}{\lambda}, \qquad \beta = \tilde{g} + \frac{g\xi}{\lambda}, \qquad \gamma = \sqrt{\frac{2\alpha}{\xi}}. \tag{5}$$

Note that the parameter $\beta$ is invariant under an arbitrary shift in $\varphi$: $\varphi \to \varphi + \delta\varphi$.

Making the conformal transformation of the metric (not of the scalar field)

$$g_{\mu\nu} \;=\; \tilde{g}_{\mu\nu} e^{2\rho}, \qquad e^{2\rho} = 1 + B\,, \tag{6}$$

after some algebra we arrive at the action

$$S_0 = \int d^4x \sqrt{-\tilde{g}} \left\{ \frac{M_P^2}{2} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - U_0(\chi) \right\}, \tag{7}$$

and the minimal potential term is

$$U_0(\chi) \;=\; \frac{M_P^4}{8\alpha}\, \sigma^2 B^2, \tag{8}$$

written in terms of the variables

$$\sigma = \sigma(\chi) = e^{-\sqrt{\frac{2}{3}}\frac{\chi}{M_P}}, \quad B = B(\chi) = e^{\sqrt{\frac{2}{3}}\frac{\chi}{M_P}} - 1. \tag{9}$$

This potential is the Einstein-frame mapping of the $R + R^2$ action of the Starobinsky inflationary model.

As the next step, consider the first order in perturbations. Starting from the modified version of the change of variables (3), we get

$$M_P^2 B \;=\; \xi\varphi^2 - 2\tilde{g}\varphi. \tag{10}$$

Replacing $\varphi = \varphi_0 + \varphi_1$ into this equation, in the first order in $\tilde{g}$, we get

$$\varphi^2 \;=\; \frac{M_P^2 B}{\xi} \;+\; \frac{2\tilde{g} M_P B^{1/2}}{\xi^{3/2}}. \tag{11}$$

The action in terms of the field $B$ has the form

$$\begin{aligned} S_1 \;=\; \int d^4x \sqrt{-g} \bigg\{ & \frac{M_P^2}{2} R(B+1) \\ & - \frac{\lambda M_P^4 B^2}{6\xi^2} - V_1(B) \bigg\}, \end{aligned} \tag{12}$$

where

$$\begin{aligned} V_1(B) \;=\; & \frac{m^2 M_P^2 B}{2\xi} \;+\; \frac{g M_P^3 B^{3/2}}{\xi^{3/2}} \\ & + \frac{\tilde{g}\lambda M_P^3 B^{3/2}}{6\xi^{5/2}} \;-\; \frac{\tau M_P B^{1/2}}{\xi^{1/2}}. \end{aligned} \tag{13}$$

Before making the conformal transformation, let us write the action (12) in terms of a more useful notation. From now on, we shall express the parameters in units of the (reduced) Planck mass $M_P = (8\pi G)^{-1/2}$, that is

$$m' = \frac{m}{M_P}, \quad \tau' = \frac{\tau}{M_P^3}, \quad \tilde{g}' = \frac{\tilde{g}}{M_P}, \quad g' = \frac{g}{M_P}. \tag{14}$$

In these units, according to what we have discussed above, $m'$, $|\tau'|$, $|\tilde{g}'|$, $|g'| \ll 1$.

It is important that the conformal transformation is not affected by the small perturbation terms, and should have the same form (9) is in the unperturbed version of the theory. The reason is that the curvature $R$ enters Eq. (12) in exactly the same form as in the action (4), so this is the unique form of conformal transformation providing the canonical kinetic term for the scalar $\chi$.

Dropping the primes, the new action has the form (7) with the full potential of the form

$$\begin{aligned} U(\chi) = & \frac{M_P^4}{8\alpha}\sigma^2 B^2 + \frac{M_P^2 m^2}{2\xi}\sigma^2 B \\ & + \frac{\sqrt{2}\gamma\beta M_P^4}{4\alpha^{3/2}}\sigma^2 B^{3/2} - \frac{\tau\gamma M_P^4}{\sqrt{2\alpha}}\sigma^2 B^{1/2}, \end{aligned} \tag{15}$$

As it should be expected, Eq. (15) includes the potential corresponding to the $R+R^2$ model, plus a perturbation.

It is remarkable that the small parameters $g$, $\lambda$, $\tilde{g}$, and $\tau$ and the parameters $\lambda$ and $\xi$, enter the expression (15) only in the combinations $\alpha$, $\beta$, $\gamma$ and $\tau$, the first three defined in (5). The scalar field $\chi$ combine into the quantities defined in (9).

The derivative of the above potential (15) is the following:

$$\begin{aligned} U'(\chi) = & \frac{M_P^3}{2\sqrt{6}\alpha}\sigma^2 B + \frac{M_P m^2}{\sqrt{6}\xi}\sigma(2\sigma - 1) \\ & + \frac{\gamma\beta M_P^3}{4\sqrt{3}\alpha^{3/2}}\sigma B^{1/2}(3 - 4\sigma B) \\ & - \frac{\tau\gamma M_P^3}{2\sqrt{3}\alpha}\frac{\sigma}{B^{1/2}}(1 - 4\sigma B) \ . \end{aligned} \tag{16}$$

In this expression, the first term is the usual one in the Starobinsky model. Now, for $\chi \to \infty$, we have that $\sigma \to 0$. Noting that $\sigma B = 1 - \sigma$ and keeping the leading orders in $\sigma$, in this limit we have then:

$$\begin{aligned} U'(\chi) \sim & \frac{M_P^3}{2\sqrt{6}\alpha}\sigma - \frac{M_P m^2}{\sqrt{6}\xi}\sigma \\ & - \frac{\gamma\beta M_P^3}{4\sqrt{3}\alpha^{3/2}}\sigma^{1/2} + \frac{3\tau\gamma M_P^3}{2\sqrt{3}\alpha}\sigma^{3/2} \,. \end{aligned} \tag{17}$$

It is easy to see that $U' \to 0$ in the limit $\chi \to \infty$. Indeed, there are various contributions, the first term being of the same order $\sigma$ as for the Starobinsky model, plus the third one, which is of order $\sigma^{1/2}$ and, therefore, dominant for the large number of e-folds $N$ counted from the end of inflation (of course, not taking into account the smallness of the parameters in the coefficients). The last one which is subdominant for $N \gg 1$. However, it should not exceed the first term at $N \sim 1$ when inflation ends in order to avoid premature end of inflation compared to the unperturbed model with $\beta = \tau = 0$. The resulting strong upper limits on these coefficients will be derived in Sec. IV.

## III. INDUCED ACTION OF GRAVITY WITH ODD TERMS

Another way of obtaining the results of the previous section, which is even simpler in fact, is to use the possibility of the approximate representation of the theory (2) with $\xi \gg 1$ as $f(R)$ gravity in the same Jordan frame (i.e. without a conformal transformation) up to small terms $\propto \xi^{-1}$. This possibility follows already from the fact that the effective Brans-Dicke parameter $\omega_{BH}$ is very small ($\approx \frac{1}{4\xi}$) for this theory while it is exactly zero for $f(R)$ gravity. The alternative derivation presented below demonstrates the possibility to avoid conformal transformation and to work directly in the Jordan frame all the time. It is useful in the case of large fermion masses since neither particle rest masses, nor the physical values of the Hubble function $H(t)$ are invariant under the conformal transformation.

Our strategy will be as follows. We perform mapping of the scalar theory with the potential (1) strongly coupled to the Ricci scalar $R$ to the form of modified $f(R)$ gravity

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} f(R). \tag{18}$$

We shall describe it here in more details than in [3]. After that the analysis of consequences for inflation, for the odd terms in the action of original scalar, becomes trivial and can be done either directly in the physical (Jordan) frame or, after the conformal transformation, in the Einstein frame, see e.g. [15] where this procedure is used for a wide class of models.

Let us start with the potential (1) and, as in the previous section, assume that the main non-minimal term $\frac{\xi}{2}R\varphi^2$ and the interacting term $\frac{\lambda}{4!}\varphi^4$ are dominating over other terms, which are regarded small corrections. The effects of these small terms using perturbations. The kinetic term in the classical action of scalar field $\varphi$, will be simply neglected. This approximation corresponds to the part of the inflationary epoch, when the potential term dominates. As we shall see in what follows, this approximation provides the mapping of the scalar potential to the $R + \alpha R^2/M_P^2$ action with a sufficiently large dimensionless coefficient $\alpha$. The known fact is that this theory fits well with the observations, justifying the approximation.

Without the kinetic term (taking this term into account leads to the non-localities, which were discussed in [16, 17]), from Eq. (1), follows

$$V'(\varphi) = m^2\varphi + \frac{\lambda\varphi^3}{6} + \frac{g\varphi^2}{2} + \tau + \tilde{g}R - \xi\varphi R = 0. \tag{19}$$

Let us solve Eq. (19) perturbatively, in the first order in the small parameters $\tau$, $\tilde{g}$ and $g$. It is useful to separate the potential in two parts, $V(\varphi) = V_0(\varphi) + V_1(\varphi)$, where

$$\begin{aligned} V_0(\varphi) &= \frac{1}{2}m^2\varphi^2 + \frac{\lambda}{4!}\varphi^4 - \frac{1}{2}\xi R\varphi^2, \\ V_1(\varphi) &= \frac{g}{3!}\varphi^3 + \tau\varphi + \tilde{g}R\varphi. \end{aligned} \tag{20}$$

For the sake of generality, we keep the mass-dependence exact until the end of the consideration. The zero-order reduction of (19) has the form

$$\begin{aligned} V_0'(\varphi) &= m^2\varphi_0 + \frac{\lambda}{6}\varphi_0^3 - \xi\varphi_0 R = 0 \\ \Longrightarrow \qquad \varphi_0^2 &= \frac{6}{\lambda}\big(\xi R - m^2\big). \end{aligned} \tag{21}$$

Substituting this result into the first-order equation, with $\varphi = \varphi_0 + \varphi_1$, after a small algebra we obtain from Eq. (19)

$$\varphi_1 = -\frac{3g}{2\lambda} - \frac{\tau + \tilde{g}R}{2(\xi R - m^2)}. \tag{22}$$

According to the simplified version of the mapping (see, e.g., [18]), the function $f(R)$, in the first order of perturbation theory, has the form

$$f(R) = R - \frac{2}{M_P^2}\Big[V_0(\varphi_0) \ + \ V_1(\varphi_0) \ + \ \varphi_1 V'(\varphi_0)\Big], \tag{23}$$

where the last term obviously vanishes. In this way, substituting (21) into the potential, we arrive at the expression

$$\begin{aligned} \frac{M_P^2}{2}f(R) = \frac{3}{2\lambda}m^4 + \left(\frac{M_P^2}{2} - \frac{3\xi}{\lambda}m^2\right)R + \frac{3\xi^2}{2\lambda}R^2 \\ + \sqrt{\frac{6}{\lambda}(\xi R - m^2)}\Big[\frac{g}{\lambda}(\xi R - m^2) + \tau + \tilde{g}R\Big]. \end{aligned} \tag{24}$$

The first term in this expression is the induced cosmological constant, the second is the Einstein-Hilbert term with an induced correction to the gravitational constant, c.f. Eq. (26), which is irrelevant under the condition $\xi m^2 \ll \lambda M_P^2$ mentioned above. The third term is the $R^2$, which is an important element of the inflationary model of [13]. According to the standard evaluation [14], the magnitude of the coefficient $\frac{3\xi^2}{2\lambda}$ should be of the order $5 \times 10^8$; hence, the natural value of the main non-minimal parameter is $\xi \sim 10^4$. In the inflationary regime, $\xi R \gg \frac{\lambda M_P^2}{\xi} \gg m^2$.

All odd parameters and $m^2$ are small and their effect on the cosmic perturbations should be considered in the linear approximation. As we are interested in the odd terms, we can safely set $m^2$ to zero in Eq. (24). In this way, we arrive at the action

$$
\begin{aligned}
S = \int d^4x\sqrt{-g}\Big\{ \frac{M_P^2}{2}R - \frac{3\xi m^2}{\lambda}R + \frac{3\xi^2}{2\lambda}R^2 \\
+ M_P\sqrt{\frac{6\xi R}{\lambda}}\Big[\tau M_P^2 - \Big(\tilde{g} + \frac{g\xi}{\lambda}\Big)R\Big]\Big\}.
\end{aligned}
\tag{25}
$$

As far as $\xi m^2 \ll \lambda M_P^2$, the second term in the integrand produces only a small shift in the inverse Newton constant,

$$
M_P^2 \;\longrightarrow\; M_P^2 - \frac{6\xi m^2}{\lambda},
\tag{26}
$$

so it can be omitted and we arrive at

$$
\begin{aligned}
S = \frac{M_P^2}{2}\int d^4x\sqrt{-g}\Big[R + \frac{\alpha}{M_P^2}R^2 \\
+ \frac{2}{M_P}\sqrt{\frac{6\xi R}{\lambda}}\,\big(M_P^2\tau - \beta R\big)\Big],
\end{aligned}
\tag{27}
$$

We note that, due to the odd terms in the potential (1), the resulting function $f(R)$ has an unusual form with the non-integer powers $\frac{1}{2}$ and $\frac{3}{2}$ of the scalar curvature.

The following observation is in order. It is clear that the term proportional to the root of scalar curvature in the gravitational action leads to an inconsistency, since in the presence of this term there is no flat metric solution to the equations for the metric. In the present case, this does not mean that the theory which we are dealing with is inconsistent. Let us remember that (25) is not the fundamental action of gravity, but only the intermediate form of a mapping of the scalar theory with the potential (1), which is valid in the inflationary epoch only, more precisely in the slow-role phase.

Indeed, the above effective action is valid only when the $R^2$ term dominates, and therefore the new contributions $R^{1/2}$ and $R^{3/2}$ can indeed be treated as perturbations. For low energies, one has $\xi \to 0$ and so the mapping from the potential (1) to the action (25) cannot be performed. This allows us to avoid a possible disruption of the graceful exit or effects such as strong particle production or tachyonic instabilities, see e.g. [19].

If one aims to consider action (25) as a fundamental theory, valid for all $R$, then many conditions and requirements apply for its viability, as discussed extensively e.g. in Ref. [20]. For example, one must have $f'(R) > 0$ and $f''(R) > 0$ in order to guarantee that gravity is an attractive force and in order to avoid ghosts, and these requirements put constraints on the parameter space $(\xi, \lambda, \tau, \gamma, g)$. In the present case, these constraints do not apply because (25) is not regarded as a fundamental action, but only as an intermediate stage of the mapping of the scalar theory with the potential (1) to the minimal scalar model. Furthermore, according to the analysis of Ref. [20], one must extend a $f(R)$ theory to negative values of $R$ in order to guarantee a graceful exit from the inflationary case. As it stands, the $f(R)$ theory of Eq. (25) has not this extension because of the $\sqrt{R}$ term, and thus can only be regarded as an effective theory for large $R$.

Further analysis will be based on the action (27), that can be regarded as a particular case of the $f(R)$ theory (18). This action can be mapped to the usual scalar-metric action (see, e.g., [18] and further references therein), in the Jordan frame:

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \big[\phi R - V(\phi)\big], \tag{28}$$

where

$$\begin{aligned} \phi \,=\, f'(R) \,=\, 1 \,+\, \frac{2\alpha}{M_P^2} R \,+\, \gamma \left( \frac{\tau M_P}{\sqrt{R}} \,-\, \frac{3\beta}{M_P} \sqrt{R} \right), \\ V(\phi) = \phi R - f(R). \end{aligned} \tag{29}$$

Here the prime denotes derivation with respect to $R$ and we used notation (5). As before, we assume all “odd” parameters to be small and perform all calculations perturbatively, in the first order in these parameters. This approach simplifies the general procedure of [18]. Let us call $R_0$ the solution without odd terms and $\Delta R_1$ the first order correction to it. Writing $R$ as

$$R = R_0 + \Delta R_1 \tag{30}$$

where $\left|\Delta R_1 \sim O^{(1)}(\tau,\beta)\right| \ll \left|R_0\right|$, we arrive at

$$
\begin{aligned}
R^{1/2} &\approx R_0^{1/2}\left(1+\frac{\Delta R_1}{2R_0}\right), \\
R^{-1/2} &\approx R_0^{-1/2}\left(1-\frac{\Delta R_1}{2R_0}\right).
\end{aligned} \tag{31}
$$

Solving Eq. (29), at the zero order we get

$$
\phi = 1+\frac{2\alpha}{M_P^2}R_0 \quad \Longrightarrow \quad R_0 = \frac{M_P^2}{2\alpha}(\phi-1) \tag{32}
$$

and at the first order

$$
\Delta R_1 \;\approx\; \frac{3M_P\beta\gamma R_0^{1/2} - M_P^3\tau\gamma R_0^{-1/2}}{2\alpha}. \tag{33}
$$

The explicit form of the solution is

$$
\begin{aligned}
R(\phi) = \frac{dV(\phi)}{d\phi} &= \frac{M_P^2}{2\alpha}(\phi-1) \qquad (34)\\
&+ \frac{3\sqrt{2}M_P^2}{4}\frac{\beta\gamma}{\alpha^{3/2}}(\phi-1)^{1/2} - \frac{\tau\gamma M_P^2}{\sqrt{2\alpha}}(\phi-1)^{-1/2}.
\end{aligned}
$$

Finally, after integration, we obtain the potential

$$
\begin{aligned}
V(\phi) \;=\; &\frac{M_P^2}{4\alpha}(\phi-1)^2 + \frac{\sqrt{2}M_P^2}{2}\frac{\beta\gamma}{\alpha^{3/2}}(\phi-1)^{3/2} \\
&- \frac{2\tau\gamma M_P^2}{\sqrt{2\alpha}}(\phi-1)^{1/2}.
\end{aligned} \tag{35}
$$

It is useful to work with the action of the standard form, in the Einstein frame:

$$
S = \int d^4x\sqrt{-g}\left\{\frac{M_P^2}{2}R - g^{\mu\nu}\partial_\mu\chi\,\partial_\nu\chi - U(\chi)\right\}. \tag{36}
$$

Since the terms originating from the odd terms in Eq. (1) contribute only to the potential part of the action, after the Weyl transformation we get the usual relation between the field $\chi$ with canonically normalized kinetic term and the scalar $\phi$,

$$
U(\chi) = \frac{M_P^2}{2\phi^2}\,V(\phi(\chi)), \quad \text{where} \quad \phi(\chi) = e^{\sqrt{\frac{2}{3}}\frac{\chi}{M_P}}. \tag{37}
$$

After a small algebra, we find for the potential the expression (15). In the $m^2=0$ approximation, it boils down to

$$
\begin{aligned}
U(\chi) = &\frac{M_P^4}{8\alpha}\sigma^2 B^2 + \frac{\sqrt{2}M_P^4}{4}\frac{\beta\gamma}{\alpha^{3/2}}\,\sigma^2\,B^{3/2} \\
&- \frac{M_P^4\tau\gamma}{\sqrt{2\alpha}}\,\sigma^2\,B^{1/2},
\end{aligned} \tag{38}
$$

The two new terms $R^{1/2}$ and $R^{3/2}$, with the corresponding two corrections in the above potential, therefore modify the dynamics of inflation. Which of the two dominates depends on the parameters, but if these are of the same order, then the $R^{3/2}$ term dominates. Gravitational vacuum polarization from massive fermions (i.e. with masses $m \gg H$) during inflation provides a contribution $\sim R^3/M_P^2$ [3] (see also [21] and also [22] for more examples), but, owing to the Planck suppression, this is small compared to the new terms due to fermions. See also Ref. [23] in connection with RG corrections to $\xi$ resulting in its running which transforms to the running of the $R^2$ coefficient in the $f(R)$ representation.

## IV. DERIVATION OF THE SLOW-ROLL PARAMETERS

In the previous sections we derive the potential (15) in the Einstein frame from the original potential (1). Now we are in a position to use this expression to obtain the parameters characterizing the inflation.

It is useful to derive the first and second derivatives of the potential, which have the form

$$U'(\chi) = \frac{\beta\gamma M_P^3 \sigma B^{1/2}}{4\sqrt{3}\alpha^{3/2}}(3 - 4\sigma B) + \frac{\sqrt{2}M_P^3\,\sigma^2 B}{4\sqrt{3}\alpha} - \frac{\tau\gamma M_P^3}{2\sqrt{3\alpha}B^{1/2}}\left(\sigma - 4\sigma^2 B\right),$$

$$\begin{aligned} U''(\chi) \;=\; & \frac{M_P^2}{6\alpha}\left(2\sigma^2 - \sigma\right) \\ & + \frac{\sqrt{2}\beta\gamma M_P^2}{6\sqrt{\alpha^3 B}}\left(4\sigma^2 B^2 - \frac{9}{2}\sigma B + \frac{3}{4}\right) \\ & - \frac{\sqrt{2}M_P^2\tau\gamma}{3\sqrt{\alpha B^3}}\left(4\sigma^2 B^2 - \frac{3\sigma B}{2} - \frac{1}{4}\right). \end{aligned} \tag{39}$$

As in the general case (see e.g. [15]), in the slow-roll approximation the slow-roll parameters are related to the scalar field potential as follows (see e.g. [24]):

$$\epsilon = \frac{M_P^2}{2}\left[\frac{U'(\chi)}{U(\chi)}\right]^2, \quad \eta = \frac{M_P^2\,U''(\chi)}{U(\chi)}. \tag{40}$$

Keeping only the $O^{(1)}(\tau,\beta)$ terms, we get

$$\epsilon = \frac{4}{3B^2} + \frac{8\beta\gamma\,(3-4\sigma B)}{3\sigma\sqrt{2\alpha B^5}} - \frac{8\tau\gamma\sqrt{2\alpha}\,(1-4\sigma B)}{3\sigma B^{7/2}},$$

$$\eta = \frac{4(2\sigma-1)}{3B^2\sigma} + \frac{\sqrt{2}\beta\gamma\,(16B^2\sigma^2-18B\sigma+3)}{3\sigma^2\sqrt{\alpha B^5}}$$

$$- \frac{2\tau\gamma\sqrt{2\alpha}\,(16B^2\sigma^2-6\sigma B-1)}{3\sigma^2\sqrt{B^7}}. \tag{41}$$

Deep in the inflationary regime, that is, for large values of $\chi$, one can take only the leading term in each expression,

$$\begin{aligned}\epsilon &\simeq \frac{4}{3}e^{-2\sqrt{\frac{2}{3}}\frac{\chi}{M_P}} - \frac{4\sqrt{2}}{3}\frac{\beta\gamma}{\sqrt{\alpha}}e^{-\frac{3}{2}\sqrt{\frac{2}{3}}\frac{\chi}{M_P}} \\ &\quad + 8\sqrt{2}\tau\gamma\sqrt{\alpha}e^{-\frac{5}{2}\sqrt{\frac{2}{3}}\frac{\chi}{M_P}}, \\ \eta &\simeq -\frac{4}{3}e^{-\sqrt{\frac{2}{3}}\frac{\chi}{M_P}} + \frac{\sqrt{2}}{3}\frac{\beta\gamma}{\sqrt{\alpha}}e^{-\frac{1}{2}\sqrt{\frac{2}{3}}\frac{\chi}{M_P}} \\ &\quad - 6\sqrt{2}\tau\gamma\sqrt{\alpha}e^{-\frac{3}{2}\sqrt{\frac{2}{3}}\frac{\chi}{M_P}},\end{aligned} \tag{42}$$

or

$$\epsilon = \frac{4}{3}\sigma^2 - \frac{4\beta\gamma\sqrt{2\sigma^3}}{3\sqrt{\alpha}} + 8\tau\gamma\sqrt{2\alpha\sigma^5}. \tag{43}$$

$$\eta = -\frac{4\sigma}{3} + \frac{\beta\gamma\sqrt{2\sigma}}{3\sqrt{\alpha}} - 6\tau\gamma\sqrt{2\alpha\sigma^3}. \tag{44}$$

To calculate the number of e-folds, we express $\epsilon$ as a sum $\epsilon = \epsilon_0 + \delta\epsilon_1$, where $\epsilon_0 \sim O^{(0)}(\tau,\beta)$ and $\delta\epsilon_1 \sim O^{(1)}(\tau,\beta)$,

$$N(\chi) = \frac{1}{M_P}\int_{\chi_{end}}^{\chi}\frac{d\chi'}{\sqrt{2\epsilon(\chi')}}. \tag{45}$$

Expanding the square root in the integrand, leads to

$$N(\chi) = -\frac{3}{4}\int_{\sigma_{end}}^{\sigma}\frac{d\sigma'}{\sigma'^2}\left(1 + \frac{\beta\gamma}{\sqrt{2\alpha\sigma'}} - 3\tau\gamma\sqrt{\frac{2\alpha}{\sigma'}}\right), \tag{46}$$

where

$$d\sigma' = -\sqrt{\frac{2}{3}}\frac{\sigma' d\chi'}{M_P}. \tag{47}$$

After integration and using the condition $\chi_{end} \ll \chi$ (*i.e.* assuming that $\chi$ is deep in the inflationary era), we find

$$N(\chi) = \frac{3}{4}\left[\sigma^{-1} + \frac{\sqrt{2}\beta\gamma}{3\sqrt{\alpha\sigma^3}} - 6\sqrt{2\alpha}\tau\gamma\sigma^{-1/2}\right]. \tag{48}$$

We can expand on $\sigma$ in Eq. (48) to find the field $\chi$ in terms of the number of e-folds:

$$\begin{aligned}\sigma = \sigma_0 + \delta\sigma_1 = \frac{3}{4N} + \frac{\sqrt{2}}{3}\frac{\beta\gamma}{\sqrt{\alpha}}\left(\frac{3}{4N}\right)^{1/2} \\ - 6\sqrt{2\alpha}\tau\gamma\left(\frac{3}{4N}\right)^{3/2}.\end{aligned} \tag{49}$$

Then, by plugging it in Eqs. (43) and (44), and keeping only the terms up to $O^{(1)}(\tau,\beta)$, we get

$$\begin{aligned}\epsilon &= \frac{3}{4N^2} - \frac{\sqrt{6}}{6}\frac{\beta\gamma}{\sqrt{\alpha}}\frac{1}{N^{3/2}} - \frac{9\sqrt{6}}{4}\tau\gamma\sqrt{\alpha}\frac{1}{N^{5/2}}, \\ \eta &= -\frac{1}{N} - \frac{\sqrt{6}}{18}\frac{\beta\gamma}{\sqrt{\alpha}}\frac{1}{N^{1/2}} + \frac{3\sqrt{6}}{4}\frac{\tau\gamma\sqrt{\alpha}}{N^{3/2}}.\end{aligned} \tag{50}$$

The main inflationary observables are the scalar spectral index $n_s$ and the tensor-to-scalar ratio $r$ (see e.g. [24]), whose expressions in terms of the slow-roll parameters are:

$$n_s - 1 = -6\epsilon + 2\eta, \qquad r = 16\epsilon. \tag{51}$$

These observables have been constrained by the Planck mission [25] to $n_s = 0.9649 \pm 0.0042$ at 68% CL and $r_{0.002} < 0.056$ at 95% CL.

Therefore, we arrive at

$$\begin{aligned}\Delta_s^2 &= \frac{1}{144\pi^2\alpha}N^2\left(1 + \frac{2\sqrt{6}\beta\gamma}{9\sqrt{\alpha}}N^{1/2} + \frac{3\sqrt{6\alpha}\tau\gamma}{N^{1/2}}\right), \\ n_s - 1 &= -\frac{2}{N} - \frac{\sqrt{6}\beta\gamma}{9\sqrt{\alpha}N^{1/2}} + \frac{3\sqrt{6\alpha}\tau\gamma}{2N^{3/2}}, \\ r &= \frac{12}{N^2} - \frac{8\sqrt{6}\,\beta\gamma}{3\sqrt{\alpha}N^{3/2}} - \frac{36\sqrt{6\alpha}\tau\gamma}{N^{5/2}},\end{aligned} \tag{52}$$

where $\Delta_s^2$ is the scalar dimensionless power spectrum, obtained by using standard textbook formulas. See e.g. Ref. [26]. In terms of the original parameters,

$$\begin{aligned}\Delta_s^2 &= \frac{\lambda}{432\pi^2\xi^2}N^2\left[1 + \frac{4\sqrt{3}}{9\sqrt{\xi}}\left(\tilde{g} + \frac{g\xi}{\lambda}\right)N^{1/2} + \frac{18\sqrt{3}}{N^{1/2}}\frac{\tau\xi^{3/2}}{\lambda}\right], \\ n_s - 1 &= -\frac{2}{N} - \frac{2\sqrt{3}}{9\sqrt{\xi}N^{1/2}}\left(\tilde{g} + \frac{g\xi}{\lambda}\right) + \frac{9\sqrt{3}}{N^{3/2}}\frac{\tau\xi^{3/2}}{\lambda}, \\ r &= \frac{12}{N^2} - \frac{16}{\sqrt{3\xi}N^{3/2}}\left(\tilde{g} + \frac{g\xi}{\lambda}\right) - \frac{216\sqrt{3}}{N^{5/2}}\frac{\tau\xi^{3/2}}{\lambda}.\end{aligned} \tag{53}$$

Now we can obtain inequalities which the parameters $\beta$ and $\tau$ should satisfy in order for our perturbative approach to be valid. The smallness of the perturbations requires that the first correction to $\Delta_s^2$ must be $\ll 1$ both at $N \gg 1$, corresponding to the present cosmological scales, and at $N \sim 1$ when inflation ends. As it was mentioned above, the first condition restricts $\beta$, while the second one concerns $\tau$.

We can identify

$$\bar{N} \approx \left(\frac{9\sqrt{\alpha}}{2\sqrt{6}\beta\gamma}\right)^2 \tag{54}$$

as the limiting value of the $e$-fold number for which our perturbative approach makes sense for $N \gg 1$ when the correction term $\propto R^{3/2}$ dominates. In order to be consistent with the inflationary paradigm, we need $\bar{N} \gg 55$. This leads to the condition $\sqrt{\alpha} \gg 4|\beta|\gamma$, or $4|\beta| \ll \sqrt{\xi}$. Note also for generality that for a very large $N$ and $R$, when the perturbative approach does not hold true anymore and the gravitational action is dominated by the $R^{3/2}$ term , slow-roll inflation still can happen, but spacetime evolves to larger values of curvature, i.e., no exit to a low-curvature universe is possible.

On the other hand, the condition that the end of inflation should occur approximately at the same value of $R$ (corresponding to $N \sim 1$ in Eq. (45)) produces the strong restriction of the parameter $\tau$: $\sqrt{\alpha}\gamma|\tau| \ll 1$, or $|\tau| \ll \lambda\xi^{-3/2}$.

Eqs. (53) are among the main results of this paper. The different dependence on $N$ (from the observational point of view, $N = const - \ln k$ where $k$ is the inverse present scale of perturbations) in various terms in Eqs. (53) provides a remarkable possibility to distinguish between the contributions from $\tau$ and the combination $\tilde{g} + \frac{g\xi}{\lambda}$, would the scale dependence $n_s - 1(N)$ be measured with sufficient accuracy.

We have derived the results (53) in the Einstein frame, but as long as we retain up to first order corrections, they are valid also in the Jordan one, although the number of e-folds $N$ becomes a different function of the present wave vector modulus $k$. Moreover, the results (53) can be derived directly in the Jordan frame, without the conformal transformation to the Einstein frame, by using the formulas presented in Ref. [26] where a generic $f(R) = A(R)R^2/M_P^2$ model, with $A(R)$ slowly varying, is explored. Note that for the case investigated in the present paper, cf. Eq. (27), the corresponding $A(R)$ function is:

$$A(R) = \frac{M_P^2}{R} + \alpha + 2M_P\sqrt{\frac{6\xi}{\lambda R^3}}\left(M_P^2\tau - \beta R\right). \tag{55}$$

So, it is slowly varying for large values of $R$ for which the slow-roll inflationary regime is realized in this model.

In order to estimate the value of the inflationary observables coming from the new terms we need the value of the parameters $\xi$, $\lambda$, $\tilde{g}$, $g$, $\tau$. If $\lambda = 1$, we can set $\xi \approx 1 \times 10^4$, so that we recover Starobinsky inflation when $\tilde{g} = g = \tau = 0$. The value of $\tilde{g}$, $g$ and $\tau$ depends on the mass of the fermions to which the scalar is coupled, via the renormalization group equations, as explained in [3]. Without repeating the corresponding arguments, we just mention that the quantum contributions to the dimensional parameters $\tilde{g}$, $g$ and $\tau$ from a fermion loop are proportional to the respective power of the mass of a fermion.

For the first evaluation let's assume the fermion masses at the upped bound of the Standard Model, with $m_f \sim 1\,\text{TeV}$. Then, according to the previous considerations, $g = \tilde{g} \approx 1\,\text{TeV}$ and $\tau \approx 10^3\,\text{TeV}$. For these values of the parameters and $N = 60$ we find:

$$n_s = 0.965417, \qquad r = 0.003333, \tag{56}$$

and for $N = 50$:

$$n_s = 0.9582, \qquad r = 0.0048, \tag{57}$$

which are the same, to this precision, to the $R^2$ case. If we increase the magnitude of $m_f$ to the GUT's scale, for $m_f \approx 10^{14}$ GeV we find $n_s = 0.965374$ and $r = 0.003313$. For the supersymmetric GUT models, with $m_f \approx 10^{16}\,\text{GeV}$, there is $n_s = 0.961231$ (a difference of 0.43% from $R + R^2$ model) and $r = 0.001332$ (a difference of 60% from $R + R^2$ model).

We can compare the contribution to the inflationary observables coming from the induced action (27) with the second order corrections coming from pure Starobinsky inflation. The next-to-leading order contributions to $\epsilon$ and $\eta$ are, when $N = 60$:

$$\epsilon_{NL} = \frac{9}{8N^3} = 5.21 \times 10^{-6}, \tag{58}$$

$$\eta_{NL} = -\frac{3}{2N^2} = -4.17 \times 10^{-4}. \tag{59}$$

For $m_f \approx 1$ TeV we have that the corrections for $\epsilon$ and $\eta$ from the $\sqrt{R}$ and $R^{3/2}$ terms, when $N = 60$, are $\epsilon_{\text{odd}} = -1.24 \times 10^{-17}$ and $\eta_{\text{odd}} = -2.48 \times 10^{-16}$, and are indeed smaller than the next-to-leading order corrections of the $R + R^2$ model. However, if $m_f$ is as large as $m_f \approx 10^{16}\,\text{GeV}$, the new contributions to $\epsilon$ and $\eta$ are $\epsilon_{\text{odd}} = -1.25 \times 10^{-4}$ and $\eta_{\text{odd}} =$

$-2.47 \times 10^{-3}$ become more relevant than the second order corrections for the Starobinsky model.

In figure (IV) we can see the potential of Eq. (15) for a few different magnitudes of $m_f$, together with the pure Starobinsky model. It is worth noting that the values of the three parameters corresponding to the odd terms in (1) have been set to their maximal possible values. This corresponds to the usual practise of searching for upper bounds of the new parameters. Indeed, as the off parameters are all regarded small, their effects can be also explored separately. However, following the way we show the results here, the qualitative output is clear and thus, we skip the more detailed consideration. For negative values of the field, our potential becomes imaginary because of the square root. But inflation corresponds to $R > 0$ and the plateau, so in principle this does not represent a problem.

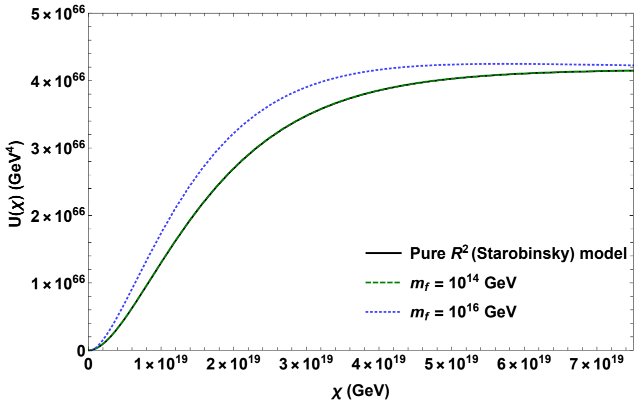


FIG. 1: *Potentials for large values of typical masses $m_f$, together with the reference plot for the $R^2$-model without odd terms. In all cases, $\xi = 10^4$.*

In order to understand better the results, at this point we have to come back to the definition of the problem in [3] and describe the physical situation in which the sterile scalar can be coupled to the Standard Model fermions.

The Standard Model left-handed fermions are doublets under the $SU(2)$ gauge group of the form $\begin{pmatrix} u_i^L \\ d_i^L \end{pmatrix}$ in the case of left-handed quarks. We have $u_i^L = \{u_L, t_L, c_L\}$ and $d_i^L = \{d_L, b_L, s_L\}$. On the other hand, the right-handed fermions are singlets under $SU(2)$

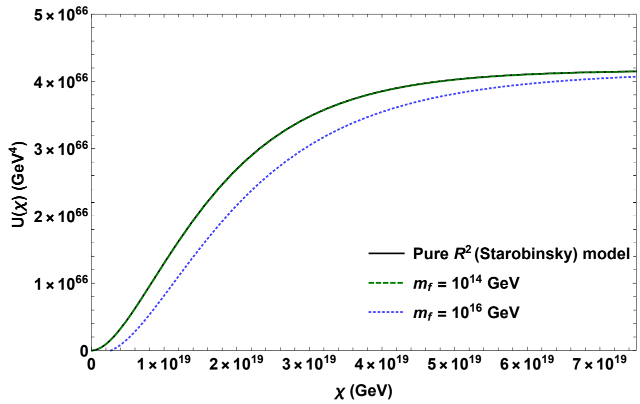


FIG. 2: *Potentials for different large values of typical masses* $m_f$*, when* $g < 0$ *and* $\xi = 10^4$*, and the reference plot for the* $R^2$*-model without odd terms. Note that the odd-terms effect is almost invisible for* $m_f = 10^{14}\, GeV$*.*

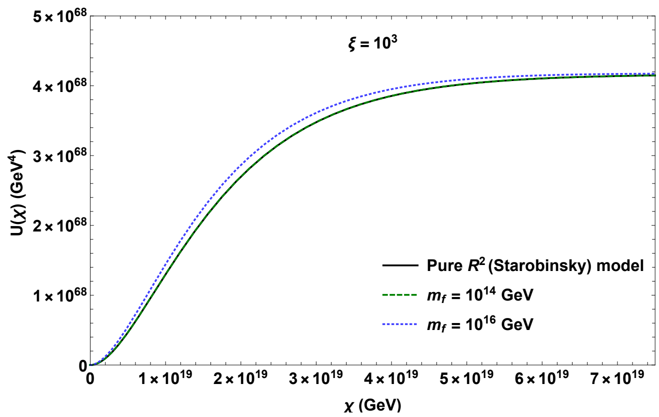


FIG. 3: *Potentials for different large values of the mass* $m_f$*, together with the reference plot for the Starobinsky model without odd terms,* $\xi = 10^3$*.*

group, namely $u_i^R$ and $d_i^R$. So, in order to have Yukawa interaction in the Lagrangian being invariant under the $SU(2)$ gauge group, the scalar must be at least a doublet under $SU(2)$, so it can multiply the $SU(2)$ index of the left-handed fermions. This is exactly why we cannot introduce directly the fermions mass terms into the Lagrangian, and we have to do so using the Higgs mechanism.

The only possibility to couple a sterile scalar to Standard Model fermions is when the

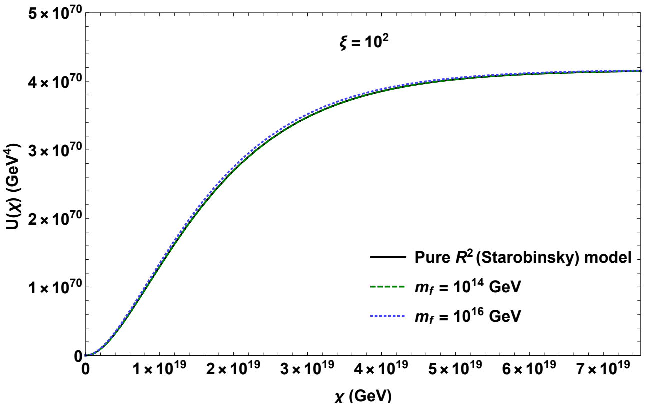


FIG. 4: *Potentials for large values of typical masse* $m_f$*, together with the reference plot for the Starobinsky model without odd terms, where* $\xi = 10^2$.

$SU(2) \times U(1)$ symmetry is broken by the Higgs mechanism, at energies below 125 GeV. At this regime, terms in the effective low-energy Lagrangian don't need to be invariant under SU(2), as this is no longer a manifest symmetry of the theory. In particular, if the sterile scalar is mixed with the Higgs at high energies, in the process of symmetry breaking the Yukawa interactions with the sterile scalar emerge in a natural way.

On the other hand, as we have seen above, the coupling of a sterile scalar (inflaton) with fermions at the Standard Model energy and mass scale does not produce essential changes in the inflationary observables. If thinking about the physics beyond the Standard Model, there may be new heavy fermion singlet fields, that could couple to a sterile scalar. Alternatively, the coupling of the inflaton with the Higgs-like scalar of GUT model can give the effect of mixing similar to the one described above for the Standard Model. This possibility gives a chance to detect the traces of GUT's in the cosmological observations.

## V. CONCLUSIONS

We have explored basic consequences of odd terms in the inflaton potential in the case when the inflaton is strongly non-minimally coupled to gravity. The presence of these odd terms is motivated by the structure of renormalization of a generic sterile scalar coupled to fermions by means of the Yukawa interaction. The analysis has been performed both by a direct transformation to the Einstein frame, and also by means of mapping to the $f(R)$ inflationary model in the original Jordan frame which has the same predictions for primordial perturbation spectra, as discussed in Sec. III. Along with an additional control, this second approach provides more intuitive understanding of the role of the odd terms.

An advantage of our approach is that the physical analysis can be performed in terms of the underlying particle physics model to which the inflaton is coupled. The values of the constants of the odd terms in the potential satisfy the lower bounds related to the running of the corresponding parameters. In practice, this means that these dimensional constants should be of at least the same order of magnitude as the heaviest fermions of the model. The main result which we obtained is that the effect of the odd terms is negligible of the typical mass $m_f$ of the heaviest fermions is smaller than the GUT scale about $10^{16}$ GeV. Thus, the odd terms in the inflaton potential become relevant only in the presence of a GUT with the corresponding fermions. If these conditions are satisfied, there is, in principle, a chance to distinguish the inflaton models from the legitimate $f(R)$ models by measuring the quantities such as the spectral index $n_s$ and tensor-to-scalar ratio $r$.

**Acknowledgments**

The authors are grateful to A. Belyaev and R. D. Matheus for the discussion of the possibility to couple sterile scalar to the Standard Model fermions. OFP thanks CNPq, CAPES and FAPES for partial financial support. The work of ISh was partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico - CNPq under the grant 303635/2018-5, , by Fundação de Amparo à Pesquisa de Minas Gerais - FAPEMIG under the grant PPM-00604-18, and by Ministry of Education of Russian Federation under the project No. FEWF-2020-0003. AAS was partly supported by the project number 0033-2019-0005 of the Russian Ministry of Science and Higher Education.

[1] J. Martin, C. Ringeval, R. Trotta and V. Vennin, *The Best Inflationary Models After Planck,* JCAP **1403** (2014) 039 [arXiv:1312.3529].

[2] L. A. Kofman, A. D. Linde and A. A. Starobinsky, *Reheating after inflation,* Phys. Rev. Lett. **73** (1994), 3195 [hep-th/9405187].

[3] V. F. Barra, I. L. Buchbinder, J. G. Joaquim, A. R. Rodrigues and I. L. Shapiro, *Renormalization of Yukawa model with sterile scalar in curved space-time,* Eur. Phys. J. C **79** (2019) 458 [arXiv:1903.11546].

[4] F. L. Bezrukov and M. Shaposhnikov, *The Standard Model Higgs boson as the inflaton,* Phys. Lett. B **659** (2008) 703 [arXiv:0710.3755].

[5] A. O. Barvinsky, A. Yu. Kamenshchik and A. A. Starobinsky, *Inflation scenario via the Standard Model Higgs boson and LHC,* JCAP **0811** (2008) 021 [arXiv:0809.2104].

[6] A. O. Barvinsky, A. Yu. Kamenshchik, C. Kiefer, A. A. Starobinsky and C. Steinwachs, *Asymptotic freedom in inflationary cosmology with a non-minimally coupled Higgs field,* JCAP **0912** (2009) 003 [arXiv:0904.1698].

[7] F. L. Bezrukov, A. Magnin and M. Shaposhnikov, *Standard Model Higgs boson mass from inflation,* Phys. Lett. B **675** (2009) 88 [arXiv:0812.4950].

[8] F. Bezrukov, A. Magnin, M. Shaposhnikov, and S. Sibiryakov, *Higgs inflation: consistency and generalisations,* JHEP **1101** (2011) 016 [arXiv:1008.5157].

[9] F. L. Bezrukov and M. Shaposhnikov, *Standard Model Higgs boson mass from inflation: Two loop analysis,* JHEP **0907** (2009) 089 [arXiv:0904.1537].

[10] T. Miranda, J. C. Fabris and O. F. Piattella, *Reconstructing a $f(R)$ theory from the $\alpha$-Attractors,* JCAP **1709** (2017) 041 [arXiv:1707.06457].

[11] R. Kallosh and A. Linde, *Universality Class in Conformal Inflation,* JCAP **1307** (2013) 002 [arXiv:1306.5220].

[12] M. He, A. A. Starobinsky and J. Yokoyama, *Inflation in the mixed Higgs-$R^2$ model,* JCAP **1805** (2018) 064 [arXiv:1804.00409].

[13] A. A. Starobinsky, *A new type of isotropic cosmological models without singularity,* Phys. Lett. B **91** (1980) 99.

[14] A. A. Starobinsky, *The perturbation spectrum evolving from a nonsingular initially de-Sitter cosmology and the microwave background anisotropy,* Sov. Astron. Lett. **9** (1983) 302.

[15] T. Miranda, C. Escamilla-Rivera, O. F. Piattella and J.C. Fabris, *Generic slow-roll and non-gaussianity parameters in $f(R)$ theories,* JCAP **1905** (2019) 028 [arXiv:1812.01287].

[16] E. V. Gorbar and I. L. Shapiro, *Renormalization Group and Decoupling in Curved Space: III. The Case of Spontaneous Symmetry Breaking*, JHEP **02** (2004) 060 [hep-ph/0311190].

[17] I .L. Shapiro, *Effective action of vacuum: semiclassical approach*, Class. Quant. Grav. **25**

(2008) 103001 [arXiv:0801.0216].

[18] D. C. Rodrigues, F. de O. Salles, I. L. Shapiro and A. A. Starobinsky, *Auxiliary fields representation for modified gravity models,* Phys. Rev. D **83** (2011) 084028 [arXiv:1101.5028].

[19] M. He, R. Jinno, K. Kamada, A. A. Starobinsky and J. Yokoyama, *Occurrence of Tachyonic Preheating in the Mixed Higgs-$R^2$ Model,* JCAP **2101** (2021) 066 [arXiv:2007.10369].

[20] S. A. Appleby, R. A. Battye and A. A. Starobinsky, *Curing singularities in cosmological evolution of F(R) gravity,* JCAP **1006** (2010) 005 [arXiv:0909.1737].

[21] I. L. Buchbinder, A. R. Rodrigues, E. A. dos Reis and I. L. Shapiro, *Quantum aspects of Yukawa model with scalar and axial scalar fields in curved spacetime,* Eur. Phys. J. **C79** (2019) 1002 [arXiv:1910.01731].

[22] I. L. Buchbinder, S. D. Odintsov and I. L. Shapiro, *Effective action in quantum gravity*, (IOP Publishing, Bristol, 1992).

[23] T. de P. Netto, A. M. Pelinson, I. L. Shapiro and A. A. Starobinsky, Eur. Phys. J. C **76** (2016) 544 [arXiv:1509.08882].

[24] O. F. Piattella, *Lecture Notes in Cosmology,* (Springer, 2018) [arXiv:1803.00070].

[25] Y. Akrami *et al. Planck 2018 results. X. Constraints on inflation* [arXiv:1807.06211].

[26] L. H. Liu, T. Prokopec and A. A. Starobinsky, *Inflation in an effective gravitational model and asymptotic safety,* Phys. Rev. D **98** (2018) 043505 [arXiv:1806.05407].