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Endurance of mesoscopic twin-beam states propagating in noisy channels

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Summary. — Quantum resources can improve the quality and security of data transmission. Here we prove, both theoretically and experimentally, that mesoscopic twin-beam states of light are robust against loss and noise sources, thus opening new perspectives in the implementation of Quantum Communication protocols.

1. – Introduction

In the field of optical Quantum Communication, quantum resources have the potential to improve both the quality and the security of transmitted information. The standard protocols employ entangled states generated at the single-photon level and the message to be transmitted is encoded in the different degrees of freedom of such states, such as polarization, momentum, time bin and orbital angular momentum [1, 2]. As an alternative, in 2002 Grosshans and Grangier proposed a different approach, based on the exploitation of continuous variables instead of discrete ones [3]. In such a case, the detection of light states is no more performed by single-photon detectors, but by means of a homodyne detection scheme.

At variance with these solutions, here we propose a different strategy, which is based on the use of mesoscopic twin-beam (TWB) states and of a commercial class of photon-number-resolving (PNR) detectors. As recently shown in [4,5], we prove that mesoscopic states of light are robust against the possible drawbacks that affect communication channels, namely loss and noise sources [6]. The use of PNR detectors operated at room temperature represents a fundamental step towards the implementation of a communication protocol in a more realistic scenario, as better discussed in the following.

2. – Theoretical model

A mesoscopic TWB state is typically multi-mode both in spectrum and space. Assuming that such modes are equally populated and that there is no phase relation among them, the resulting state can written as [7]

$$\rho_{\mathrm{TWB},\mu} = \sum_{n=0}^{\infty} P^{\mu}(n)|n,n\rangle\langle n,n|, \tag{1}$$

where $|n\rangle = \delta(n - \sum_{k=1}^{\mu} n_k) \bigotimes_{k=1}^{\mu} |n_k\rangle_k$, n_k is the number of photons in the k-th mode and n is the overall number of photons in the μ spatio-spectral modes that impinge on the detector, while $P^{\mu}(n)$ is the multi-mode thermal distribution [8]

$$P^{\mu}(n) = \frac{(n+\mu-1)!}{n!(\mu-1)!\left(\langle n\rangle/\mu+1\right)^{\mu}\left(\mu/\langle n\rangle+1\right)^{n}}, \tag{2}$$

in which $\langle n\rangle$ is the mean number of photons in the two arms of TWB. The state in eq. (1) is entangled in the number of photons. In order to prove it experimentally, it is convenient to consider the nonclassicality criterion based on the calculation of the noise reduction factor, R, which is defined as $R = \sigma^2(n_1 - n_2)/(\langle n_1\rangle + \langle n_2\rangle)$, where $\sigma^2(n_1 - n_2)$ is the variance of the distribution of the photon-number difference between the two parties, while $\langle n_1\rangle + \langle n_2\rangle$ is the shot-noise-level. The condition $R < 1$ is a sufficient condition for entanglement. In the case of the multi-mode TWB in eq. (1), R assumes the analytic form:

$$R = 1 - \frac{2\sqrt{\eta_1\eta_2}\sqrt{\langle m_1\rangle\langle m_2\rangle}}{\langle m_1\rangle + \langle m_2\rangle} + \frac{(\langle m_1\rangle - \langle m_2\rangle)^2}{\mu(\langle m_1\rangle + \langle m_2\rangle)}. \tag{3}$$

As anticipated in the Introduction, here we want to prove that mesoscopic TWB states are robust against loss and noise. To quantify their level of robustness, the expression of R can be re-calculated by assuming that there is an imbalance between the two arms of TWB, and that a noise source with mean value $\langle m_{\mathrm{N}}\rangle$ and variance σ^2_{N} is superimposed to one arm

$$R = 1 - \frac{2\eta t\langle m\rangle}{(1+t)\langle m\rangle + \langle m_{\mathrm{N}}\rangle} + \frac{(1-t)^2\langle m\rangle^2}{\mu\left[(1+t)\langle m\rangle + \langle m_{\mathrm{N}}\rangle\right]} + \frac{\sigma^2(m_{\mathrm{N}}) - \langle m_{\mathrm{N}}\rangle}{(1+t)\langle m\rangle + \langle m_{\mathrm{N}}\rangle}. \tag{4}$$

Here t is the transmission efficiency quantifying the balancing level between the two arms. According to the specific statistics of the noise source, it is interesting to investigate under which conditions it is still possible to obtain $R < 1$. As an example, in the following section we experimentally investigate the case of a quasi-single-mode thermal noise source with mean value $\langle m_{\mathrm{N}}\rangle$ and variance $\sigma^2_{\mathrm{N}} = \langle m_{\mathrm{N}}\rangle(\langle m_{\mathrm{N}}\rangle/\mu_{\mathrm{N}} + 1)$ superimposed to one arm of TWB.

3. – Experimental results

As shown in fig. 1, mesoscopic TWB states are produced by parametric down conversion in a β-Barium-Borate (BBO) crystal pumped by the fourth harmonics of a Nd:YLF laser regeneratively amplified at 500 Hz. Two portions at frequency degeneracy (523 nm) are spatially (I) and spectrally (BF) selected, focused (L) into two multi-mode fibers having 600-μm core diameter (MF) and delivered to two hybrid photodetectors (HPDs). A quasi-single-mode thermal state is produced by sending the second harmonics of the laser to a rotating ground glass disk (GD) and selecting a single speckle by means of

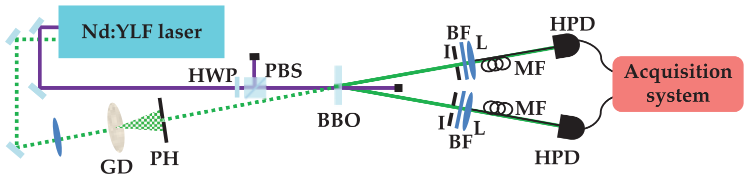

Fig. 1. – Sketch of the experimental setup. See the text for details.

a pin-hole (PH). A half wave-plate (HWP) followed by a polarizing cube beam splitter (PBS) placed on the pathway is used to change the energy of the thermal state. The thermal field is superimposed to one portion of TWB and detected together with it. Each HPD output is amplified, synchronously integrated by a boxcar-gated integrator and acquired. Also the energy of the pump field can be changed in steps by means of a HWP followed by a PBS. For each experimental condition, 100000 acquisitions are recorded. By exploiting the self-consistent method extensively explained in [9], each output of the detection chain, expressed in voltages, can be converted in number of detected photons, thus allowing to calculate all the relevant quantities to characterize the optical states. In fig. 2(a) we show R as a function of the mean value of noise for a fixed mean value

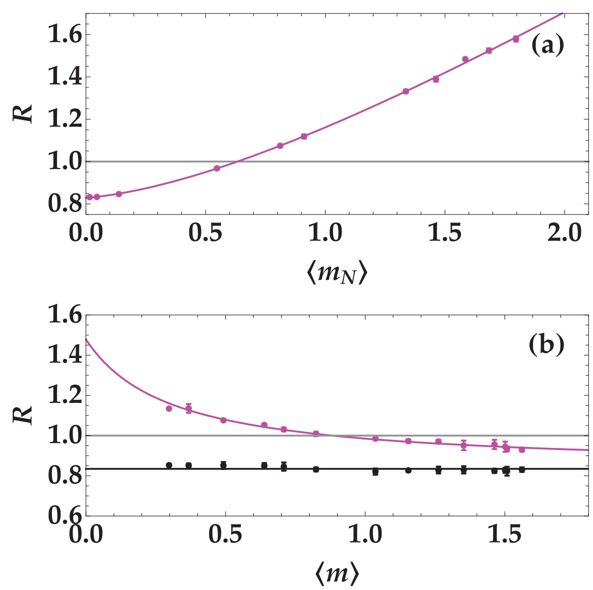

Fig. 2. – (a) R as a function of the mean number of detected photons of the thermal noise for a fixed choice of TWB. Magenta dots and error bars: experimental data; magenta curve: best fit according to eq. (4). Fitting parameters: $\langle m \rangle = 0.89$, $\mu = 49.93$, $\eta = 0.18$, $t = 0.90$, and $\mu_{\mathrm{N}} = 1.38$; χ^2_ν per degree of freedom: 2.21. (b) R as a function of the mean number of detected photons in one arm of TWB for a fixed choice of the thermal noise. Black dots and error bars: experimental data in the absence of noise; magenta dots and error bars: experimental data in the presence of thermal noise; colored curves: theoretical fitting functions according to eq. (4). From the fitting procedure applied to black dots we obtained: $\mu = 499.48$, $\eta = 0.18$, $t = 0.87$. By using the same values, from that applied to magenta dots we got $\langle m_{\mathrm{N}} \rangle = 0.57$, $\mu_{\mathrm{N}} = 1.20$. The χ^2_ν per degree of freedom is equal to 3.5 (black dots) and 3.2 (magenta dots). In both panels, the gray line at $R = 1$ represents the boundary between classical and nonclassical correlations.

of TWB, while in panel (b) we plot the same quantity as a function of the mean value of TWB for a fixed mean value of noise, both in the presence (magenta dots) and in the absence (black dots) of the noise source. It is interesting to notice that in both panels there are conditions under which it is still possible to observe nonclassical correlations. Indeed, from eq. (4) we have that $R < 1$ if

$$\langle m_{\mathrm{N}} \rangle < \sqrt{\mu_{\mathrm{N}} \left[2\eta t \mu - (1-t)^2 \langle m \rangle \right] \frac{\langle m \rangle}{\mu}}. \tag{5}$$

Hence, the maximum value of $\langle m_{\mathrm{N}} \rangle$ satisfying the nonclassicality condition is limited by the mean number of photons per mode of the TWB state, namely $\langle m \rangle / \mu$. From inequality (5) we can argue that increasing the mean value of TWB can guarantee the observation of sub-shot-noise correlations even in the presence of a thermal noise source, as far as the detection apparatus is able to properly resolve the number of detected photons.

4. – Conclusions

We have shown that mesoscopic TWB states are good candidates for encoding and transmitting information in Quantum Communication protocols, since they are robust against loss and noise. In particular, we have proved that there are conditions under which a quasi-single-mode thermal state superimposed to one arm of TWB does not prevent the observation of nonclassical correlations. Increasing the mean value of TWB could help even in the presence of a non-negligible noise source. To this aim, the use of Silicon photomultpliers instead of HPDs as PNR detectors could be crucial, since they are characterized by a wide dynamic range.

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