We present a theory of single-electron tunneling transport through a ferromagnetic nanoparticle in which particle-hole excitations are coupled to spin collective modes. The model employed to describe the interaction between quasiparticles and collective excitations captures the salient features of a recent microscopic study. Our analysis of nonlinear quantum transport in the regime of weak coupling to the external electrodes is based on a rate-equation formalism for the nonequilibrium occupation probability of the nanoparticle many-body states. For strong electron-boson coupling, we find that the tunneling conductance as a function of bias voltage is characterized by a large and dense set of resonances. Their magnetic field dependence in the large-field regime is linear, with slopes of the same sign. Both features are in agreement with recent tunneling experiments.