ISTITUTO LOMBARDO - ACCADEMIA DI SCIENZE E LETTERE

RENDICONTI

CLASSE DI LETTERE E SCIENZE MORALI E STORICHE

Vol. 139
(2005)

ESTRATTO

Flavia Cortellezzi

INVESTMENTS AS REAL OPTIONS

Scienze economiche

Istituto Lombardo (Rend. Lett.) 139, *115-150 (2005)*

INVESTMENTS AS REAL OPTIONS (*)

FLAVIA CORTELEZZI

Nota presentata dal m. e. Piercarlo Nicola
(Adunanza del 3 marzo 2005)

SUNTO. — È oggi largamente riconosciuto che i metodi di capital budgeting tradizionali non riescono a catturare alcune peculiarità della gestione dell'impresa, quali la discrezionalità del manager e/o l'importanza strategica delle decisioni di investimento. Negli ultimi due decenni, il problema è stato affrontato attraverso il "metodo delle opzioni reali", tema dominante della letteratura economico-finanziaria, ma anche ampiamente utilizzato nella valutazione delle decisioni di investimento in alternativa ad altre tecniche di valutazione di un progetto. È basato sull'esistenza di una stretta somiglianza tra un'opzione finanziaria ed una decisione di investimento: un'impresa che desidera investire in un determinato progetto detiene la possibilità di attendere le condizioni economiche a lei più favorevoli per poter realizzare l'investimento. Tale "possibilità" è assimilabile ad un'opzione finanziaria, il cui sottostante è costituito dalle variabili economiche che condizionano l'evoluzione del valore del progetto (le quote di mercato, il valore del prodotto comprato e/o venduto, l'intensità della domanda, etc.) ed il cui strike price è rappresentato dal costo di investimento. In letteratura essa prende il nome di "opzione reale" e la sua valutazione è simile a quella di altri strumenti derivati. Attraverso questo lavoro si vuole offrire una sintesi critica letteratura, nonché una visione generale delle sue possibili applicazioni.

(*) Dipartimento di Diritto ed Economia delle Persone e delle Imprese - Università dell'Insubria.

1. Introduction

In recent years, the real option theory has been widely used in evaluating investment decisions. The theory of real options was developed through the 1980's, benefitting from the wide success of financial derivatives. Real options are not only academic thought exercises, they are more and more used by corporate decision-makers, more or less consciously. As Luehrman states in the Harvard Business Review (1988):

> "*The analogy between financial options and corporate investments that create future opportunities is both intuitively appealing and increasingly well accepted. Executives readily see why investing today in R&D, or in a new marketing program, or even in certain capital expenditures (a phased plant expansion, say) can generate the possibility of new products or new markets tomorrow*".

Real options have been mentioned, among the others, in Business Week (1). For managers, real options seem to appear as superior to the traditional method of discounting future cash flow so as to decide whether to invest or not. As written in Business Week,

> "[By] *boiling down all the possibilities for the future into a single scenario, NPV doesn't account for the ability of executives to react to new circumstances – for instance, spend a little up front, see how things develop, then either cancel or go full speed*".

Moreover, in the business and economic press it is possible to find a lot of cases in which the "real option approach" has been used to asses an investment opportunity. The pharmaceutical company Merk is, for example, the first that has explicitly embraced real option pricing and that has started to use an option valuation approach in the R&D (2) resource allocation process. The company uses the stock volatility of a representative biotechnology firm to approximate the volatility of the present value of future cash flow resulting from a R&D

(1) Coy Peter, *Exploiting Uncertainty: The Real Option Revolution in Decision Making*, Business Week, June 7, 1999, pp. 118-124.

(2) See Lint (2001) for a Retrospective on the importance of real option in R&D.

project in about the same pharmaceutical field. This methodology makes it possible to apply the standard models for financial option pricing to R&D option valuation. The basic assumption underlying this approach is that the risk characteristics of the single R&D project under consideration exactly replicate the risk characteristics of the representative stock (3). This is just an example. Traditionally, natural-resources companies have been among the most enthusiastic experimenters with real options, primarily because they can link the future value of their assets to traded commodities, for which market information is readily available. Rio Tinto has been experimenting with real options for more than ten years (4).

What is exactly the *real option approach*? The economic literature on real potion approach to investment under uncertainty exploits an analogy between real and financial investment decision. More precisely, whenever an opportunity to make a real investment arises, it can be considered as a call option on a stock that consists of the capital in place so that investing is like exercising the option and the cost of the investment is the strike price of the option. This approach is based on the intuition that most real investment decisions share the three following characteristics: a) there is uncertainty, i.e. the effects of the decision are uncertain; b) once taken, the investment decision is irreversible, i.e. it entails a sunk cost; and c) the decision can be postponed, i.e. it is possible to wait for a new information to arrive that might affect the desirability or the timing of the investment expenditure. As a main result, the theory offers the conclusion that, under the above assumptions, it may be better to delay the investment, even when the immediate investment has a positive net present value (NPV) (5).

(3) The write up about Merk has created a lot of attention in the business press. See, e.g., *Exploiting Uncertainty: The Real Option Revolution in Decision Making*, Business Week, June 7, 1999; *Economics Focus: Keeping All Option Open*, The Economist, August 14, 1999.

(4) CIO Enterprise Magazine, June 15, 1999.

(5) The NPV rule is based on some particular assumptions it is worth remembering. It assumes that either the investment is reversible, i.e. it can somehow be undone and the expenditures recovered if market conditions turn out to be worst than anticipated, or if the investment is irreversible, it is a now or never position.

What causes irreversibility? Irreversibility usually arises because capital is industry or firm specific, i.e. it cannot be used productively in a different industry or by different firms. For example, expenditures in marketing and advertising are obviously firm-specific. Also, a steel plant can be industry-specific since the steel industry is highly capital-intensive and the plant can only be used to produce steel. Particularly, in a competitive market, if an investment is seen as a bad investment by one company, it is quite possible for another company to view it as a bad investment. Thus, investment is not worth as much as it was. Partial irreversibility can also result from the lemon effect studied by Akerlof (1970). He argues that with the existence of quality differences and uncertainty, the Gresham's law holds in a modified reappearance: even second-hand office equipments, cars, trucks, have a resale value well below their purchase cost, even if they are new. Finally, irreversibility arises because of government regulations or institutional agreements. An investment in new workers may be partially irreversible because of high cost of firing imposed by institutions. In addition, expenditures like entry cost and recruiting cost can be so high that investments may be in large part irreversible. For instance, training costs are spent on improving the quality of human capital, which is not readily kept within the company. The second characteristic concerns the possibility to delay an investment. Firms do not always have the possibility to postpone the investment expenditure. In some circumstances it is imperative for a firm to invest quickly and thereby pre-empt investment by the existence of potential competitors. But in most cases delay is at least feasible. There may be some costs to delay: the risk of entry by other firms, or simply foregone cash flow. These costs should be weighted against the benefits deriving from the opportunity to wait for new information. These two features together make the investment particularly sensible to uncertainty over costs, profits, interest rate, market conditions and government policy.

How does this approach allow us to interpret the managers' operating decision? When undertaking any particular project, management seldom binds itself to a simple irrevocable operating strategy for that project. Should the future deviate from expectations, management has some degree of discretion regarding the operation of the project. Where this discretion exists, management may be said to hold some *operating options* whose common feature is the uncertainty underlying

assets associated with real options. By making an initial investment, the option buyer obtains a right to the resources necessary to compete in the market, but defers any further commitment until some future time, giving the possibility of waiting for new information to arrive that might affect the desirability or timing of the expenditure. View in this way, the true value of a project should consider the call option value as an opportunity cost that must be included as part of the investment cost. The subsequent decision to exercise an option involves striking the option by making the sunk investment necessary for full market entry. Another of the most significant of such operating option is the option to abandon a project earlier than originally planned. Conventional discounted cash flow techniques recognise the desirability of abandonment when salvage values equal or exceed the present value of future operating cash flow and fixes the terminal date of the project at that point. However, earlier termination may be warranted if, for example, the project fails or a change in technology requires the substitution of a new type of equipment for the old, or as future price/cost fluctuate, the operating profit of a project in place may turn be negative. The best time to abandon a project may never be known in advance with absolute certainty. But the right to abandon at any time under various circumstances is tantamount to owning a put option on the project's assets. The exercise price of the put is the salvage value that could be received on disposal of the asset. View in this way, the true value of a project is not just the present value of cash flow forecasted to some termination date, but that present value plus the value of the option to abandon. The option to convert assets from one use to another is a type of operating options that can be constructed as a special case of the abandonment option. Again, management effectively has a put option on the project's assets. The exercise price of the put option is the assets' value in their most productive use. Because conversion of use can, in principle, takes place a number of times, this operating options might best be thought of as a compound option, which is an option on a series of future options. In a similar way, the choice of whether to operate a plant or some equipment after being placed into a service represents a valuable operating option. As McDonald and Siegel (1986) pointed out, as production might be shut down and restarted at any time, the option to operate is more likely a large portfolio of European call options, each of which expires at different dates throughout the life of the assets. The underlying asset of the option to operate is the revenue that will be produced by the asset and the exercise price is the variable production cost. If revenues fall below variable production costs, then, the option to operate will be "out of the money" and will not be exercised. Finally, an important operating option discussed in Mason and Merton (1985) is the option to expand or contract the scale of operations. Many projects are engineered in such a way that output can be expanded or contracted in the future, contingent upon the market conditions. This might be achieved through planned redundant capacity or built-in-flexibility to alter the rate of output and length of production.

For most managers, the existence and the value of these various types of operating options are obvious to the point of being beyond conscious considerations. Perhaps because of this fact, they are so often neglected in conventional discounted cash flow analysis, which tends to be executed in such a way as to reflect a single, irrevocable operating strategy. Failure to reflect such operating flexibility, however, will understate the true value of the project; it should be necessary to include all relevant option values in the definition of the net present value.

Capital investment projects may be more valuable then recognised by discounted cash flow analysis not only because of operating flexibility, but also because of valuable new investment opportunities that the project may create. Provided that they are discretionary so there is no future obligation to invest capital, these follow-up projects can also be viewed as call options on assets, hereinafter referred to as growth options. By the way of analogy to ordinary call options on securities, a growth option exercise price is the investment required to obtain the assets. The value of the asset to be acquired is the present value of the expected cash flow plus the value of any growth options expected through ownership and operation of the assets. The time of maturity is the length of time that commitment of capital can safely be deferred before the opportunity ceases to exist. Clearly, this might be many years or just a few days, depending on the nature of the opportunity. Virtually any discretionary investment opportunity may be viewed as a growth option. A capacity expansion project, new product introductions, and acquisitions are other general forms of investment that can be treated as growth options. Although outlays for advertising,

basic research, and commercial development programs are typically expensed than capitalized, these programs may also be treated as growth options insofar as they represent investments in real assets like brand name and technical expertise that will generate long term benefits. Even maintenance and replacement projects are in the nature of growth options since they can be foregone if management wishes, and their undertaking is tantamount to a decision not to shrink a business or exit from it immediately. Growth options are of great importance to modern corporations, since they account for such a large fractions of a typical firm's market value.

This paper discusses several theoretical recent studies on the various implementation of the real option approach and it is organised as follows. Section 2 reviews the basic continuous-time stochastic model to illustrate how uncertainty and irreversibility may affect an investment decision and how dynamic programming can be used to value an investment opportunity and determine whether or not the firm should invest. The literature is then summarized, highlighting the building blocks of the real option literature, namely the role of different sunk costs in the operating decisions of firms, the role played by uncertainty, and the flexibility of timing. Section 3 reviews the basic model of imperfect competition and shows how they can be used to study a technology based competition. Section 4 reviews how this method can be applied to study different economic phenomena. Finally, Section 5 illustrates how the financing problem can be introduced in real option models. Section 6 concludes and highlights some avenues for future research.

2. Basic Models

This Section aims to analyze how a project and an investment opportunity can be viewed as an option, and valued accordingly, considering a complex continuous time model.

2.1. *The Investment Decision: A Simple Continuous Time Model*

The model developed by McDonald and Siegel (1986) is the most basic continuous time model of irreversible investment. In this model (6), a firm must decide "if and when" to invest in a single irreversible project. The authors pointed out that the investment decision is equivalent to a perpetual call option, and deciding when to invest is equivalent to deciding when to exercise such an option. The cost of the investment, I, is known and fixed, but the value of the project, V, follows a geometric Brownian motion (7)

$$dV = \alpha V dt + \sigma V dz \tag{1}$$

where α is the expected growth rate of the project value, σ is the standard deviation, and dz is the increment of a Wiener process,

$$dz = \varepsilon(t)\, dt^{\frac{1}{2}} \tag{2}$$

where $\varepsilon(t)$ is a serially uncorrelated and normally distributed random variable. Equation (1) implies that the current value of the project is

(6) The model here reported draws heavily on Dixit and Pindyck (1994, Chapter 4 and 5).

(7) For a clear and rigorous exposition of the theory of optimal regulation of Brownian motion see, e.g. Dixit (1991) and Moretto (1995). Assuming V to follow a geometric Brownian motion and not introducing the possibility that the firm goes bankruptcy allows the authors to develop the analysis under the Modigliani-Miller assumption and therefore avoid the problem of interaction between investment and financing decision. Moreover, all the relevant parameter are known.

known, but future values are lognormally distributed with expected value $E(V_t) = V_0 e^{\alpha t}$ and a variance that grows linearly with the time horizon, $var(V) = \sigma^2 t$. Thus, although information arrives over time (the firm observes V changing), the future value of the project is always uncertain (8). It is a special case of an Ito's continuous time stochastic process (9).

The simplest Net Present Value Rule is to invest as soon as $V > I$, but as McDonald and Siegel demonstrated, this is incorrect. Because future values of V are unknown, there is an opportunity cost to investing today. Hence, the optimal investment rule is to invest when V is at least as large as a critical value V^* that exceeds I, even by two or three times for reasonable values of the parameters. They considered the following problem: at what point is it optimal to pay a sunk cost I in return for a project whose value is V, given that V evolves according to equation (1)? The project (10) has payoffs equal to the value in the chosen period net of the investment cost, $V_t - I$. The firm will wish to time its investment decision so as to maximise the expected present value of the option to invest $F(V)$ given by:

(8) An important question concernes the source of uncertainty in the investment payoff function. One obvious source of uncertainty which manifest itself in product price fluctuations derives from future demand uncertainty. McDonald and Siegel (1986) worked out a simple example in which V is derived in terms of production and demand parameter. They also demonstrate the weakness of the assumption that θ_t follows a geometric Brownian motion. As θ_t represents a real shock it is unreasonable to expect uncertainty about future values of the stock to grow linearly with time. This is a simplifying assumption which allows to get closed form solutions. In a similar context Metcalf and Hasset (1995) showed how a more plausible stochastic process (e.g. a mean reverting process, which captures the notion that in the long run demand can be expected to be at a "normal level") gives the same results.

(9) The calculus of continuous time process is described in Dixit (1993).

(10) To evaluate an investment option using the standard methods of financial option, requires that stochastic changes in V are spanned by the existing asset, i.e. markets are complete. The contingent claims valuation can therefore be used (see Pindyck (1991) and Dixit and Pindyck (1994) for the analysis using the contingent claim methods). However there can be cases in which this assumption does not hold, e.g. the introduction of new product unrelated with the market. It can be shown (see Pindyck (1991)) that dynamic programming can still be applied.

$$F(V) = \max E_t\,[(V_T - I)e^{-rT}] \tag{3}$$

subject to equation (1) for V, where E_t denotes the expectation at time t, T is the unknown future point in time at which the investment decision is made, and r is the discount rate ([11]).

Delaying the investment decision and holding the option is equivalent to holding an asset which pays no dividends but may appreciate as time passes. The fundamental condition for optimality, or the Bellman equation, if the firm delays investment and holds the option is given by:

$$rF = \frac{E\,(dF)}{dt}\,. \tag{4}$$

The left hand side of equation (4) is the discounted normal rate of return that an investor would require for holding the option, while the right hand side is the expected total return per unit of time from holding the option. If this condition holds, then the firm is equating the expected return from delaying the investment with the opportunity cost of delay. In effect equation (4) describes a non-arbitrage condition.

Using Ito's lemma to obtain the total differential of a continuous time stochastic process, we can express dF as:

$$dF = F'(V)\,dV + \frac{1}{2}\,F''(V)\,(dV)^2\,. \tag{5}$$

Using the expression in equation (1) for dV and taking expectations gives:

$$E\,(dF) = \alpha V\,F'(V)\,dt + \frac{1}{2}\,\sigma^2 V^2\,F''(V)\,dt\,. \tag{6}$$

([11]) Note that r must be greater than α otherwise the firm will hold the option to delay forever. To correctly compute the discount rate see McDonald and Siegel (1986), pp. 715-716.

Substituting (5) into (6) we obtain the Bellman equation in the case where dV is a continuous stochastic process:

$$rF = \alpha V\,F'(V) + \frac{1}{2}\,\sigma^2 V^2\,F''(V)\,. \tag{7}$$

If the firm is following the optimal investment rule, the value of the option to wait must satisfy the second order differential equation given by (7). In addition it must satisfy three boundary conditions:

$$F(0) = 0 \tag{8}$$

$$F(V^*) = V^* - I \tag{9}$$

$$F'(V^*) = 1\,. \tag{10}$$

The first condition simply states that if the value of the investment falls to zero then the value of the option to invest is zero. The second describes the net payoff at the value of V at which it is optimal to invest. The third is the smooth pasting condition (Dixit, 1993) which require the function $F(V)$ to be continuous and smooth around the optimal investment timing point. If smooth-pasting was violated and instead a kink arose at V, a deviation from the optimal policy would raise the firm's expected payoff. By delaying for a small interval of time after the stochastic process first reached V, the next step dV could be observed. If the kink was convex, the firm would obtain a higher expected payoff by entering if and only if V has moved (strictly) above V^*, since an average of points on either side of the kink give it a higher expected value than the kink itself. If the kink was concave, on the other hand, second order conditions would be violated. Continuation along the initial value function would yield a higher payoff than switching to the alternative function and switching at V could not be optimal. More detailed explanation of this condition can be found in Dixit and Pindyck (1994), chapter 4, Appendix C. The solution to (7) subject to condition (8), (9), (10) is:

$$F(V) = \frac{(\beta_1 - 1)^{\beta_1 - 1}}{(\beta_1)^{\beta_1}\, I^{\beta_1 - 1}} \tag{11}$$

where

$$\beta_1 = \frac{1}{2} - \frac{(r-\alpha)}{\sigma^2} + \sqrt{\left[\frac{(r-\alpha)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} > 1 .$$

By substituting (11) into the second and third boundary conditions, we obtain the result that the optimal investment timing payoff is given by:

$$V^* = \frac{\beta_1}{\beta_1 - 1}\, I . \tag{12}$$

It is worth noticing some of the characteristics of this solution and the economic meanings. The most important point is that since $\beta_1 > 1$ we have that $\frac{\beta_1}{\beta_1 - 1} > 1$ and therefore $V^* > 1$. Thus, uncertainty and irreversibility drive a wedge between the critical value V^* and I ([12]). The size of the wedge is the factor $\frac{\beta_1}{\beta_1 - 1}$, called the option value multiple, and it becomes important to examine its magnitude for realistic values of the underlying parameters. Specifically, it has been proved that an increase in variance results in an increase of the option value multiple. Therefore the greater is the amount of the uncertainty over future values V the larger is the excess return the firm will demand before it is willing to make an irreversible investment ([13]). As a result, when a firm's market becomes more uncertain the stock market value of a firm can go up, even if the firm does less investing and perhaps produces less. Thus, uncertainty increases the value of a firm's investment opportunities, irrespective of investors' or managers' risk preferences, and irrespective of the extent to which the riskiness of the project's value is correlated to the market and decreases the amount of the actual investing that the firm will do. The same result applies if the interest rate r increases. The economic intuition is the reduction of the investment cost over time due to an higher discount factor. In other words, an increase in the real interest rate results in fewer investment options becoming exercised. Hence higher real interest rates reduce investment but for different reasons than in the standard model. Finally, as the expected growth rate of the value of the project increases, the lower will be the option value multiple.

([12]) According to the neoclassical theory of investment, firms should invest until the value of an incremental unit of capital is just equal to its cost (the fundamentals of this theory lie in the NPV rule). However the investment is considered fully reversible. In presence of irreversibility and uncertainty, the real option theory suggests to include the option to wait as an opportunity cost. See Marzano and Saltari (1997) for an excellent survey on investment under certainty and uncertainty.

([13]) Sarkar (2000) points out two effects of uncertainty of investment. On the one hand, in line with the real option theory, uncertainty increases the investment threshold and therefore reduces the investment. On the other hand, computing the probability of investing, i.e. the probability of reaching the critical threshold V^*, he shows that uncertainty increases the probability that the investment threshold will be hit. Therefore, higher level of uncertainty might have a positive effect on investment.

2.2. *The Building Blocks of the Real Option Theory*

In this section we shed light on the building blocks of the real option theory, namely to underlying the role of irreversibility, uncertainty and the flexibility of timing.

2.2.1. Irreversible Investment and Operating Decisions

Abandoning decision. Among the option held by a manager we identified in Section 1, there is the option for a firm operating in an existing industry to abandon a project. In this case, the owner of the project holds a put option, an option to sell the project for the net scrap value, and this raises the project's value. This has been analyzed by Mayers and Majd (1985). This creates an opportunity cost of shutting down, because the value of the project might rise in the future. There may also be sunk costs associated with the operation of the project. In fact, for most projects, there are likely to be substantial sunk costs involved in even temporarily shutting down and restarting.

Investment and abandoning options: the link. The valuation of projects and the decision to invest and abandon a project when there are sunk costs have been studied by Brennan and Schwartz (1985) and Dixit (1989a). Brennan and Schwartz (1985) study the evaluation of natural resource investments under highly volatile output prices. In the model, Brennan and Schwartz not only show how the assets, whose cash flows depend on the highly volatile output price, can be valued, but also demonstrate the effect of sunk costs on the decision to open and close (temporarily or permanently) a mine, when the price of the resource follows a geometric Brownian motion. Their work shows how sunk costs of opening and closing a mine can explain the "hysteresis" often observed in the extractive resource industries: during periods of low prices, managers often continue to operate unprofitable mines that had been opened when prices were high; at other times managers fail to reopen profitable ones that had been closed when prices were low. This insight is further developed in Dixit (1989a, 1991). Dixit (1989a) isolates the entry and exit decisions and studies a model with sunk costs I and E of investing and abandoning a project. The project produces one unit of output per period, with variable cost C. The output price P follows a geometric Brownian motion. If $\sigma = 0$, the standard result holds: enter if $P \geq C + \rho I$ and exit if $P \leq C - \rho E$ (ρ is the discount factor and σ is the standard deviation of the market considered). However, if $\sigma > 0$ there are opportunity costs to entering or exiting. Intuitions suggest an idle firm will invest when demand conditions become *sufficiently* favourable, and an active firm will abandon when they become *sufficiently* adverse. Indeed, the optimal strategy for holding or exercising the two options, will take the form of two threshold prices, P_H and P_L (H = high and L = low), with $P_H > P_L$. An idle firm will find it optimal to remain idle as long as P remains below P_H, and invests as soon as P reaches the threshold P_H. An active firm will remain active as long as P remain above P_L, but it will abandon if P falls to P_L. In the range of prices between the threshold P_H and P_L, the optimal policy is to continue with the status quo. Denoting with $V_0(P)$ the value to invest of an idle firm, and with $V_1(P)$ the value of an active firm, that is the sum of two components, the entitlement to the profit from the operation and the option to abandon should the price fall too far. Applying the usual boundary conditions it is possible to numerically derive the thresholds that trigger the investment. In particular, it is

worth noticing that the opportunity costs raise the critical price above which it is optimal to enter and lower the critical prices below which it is optimal to exit. These models help to explain the prevalence of hysteresis. In Dixit's model, a firm that entered industry when prices were high may remain there for an extended period of time even though the prices have fallen below variable cost, so they are losing money. And firms that leave an industry after a protracted period of low prices may hesitate to reenter even after prices have risen enough to make entry seem profitable. Dixit (1992) further discusses how hysteresis influence investment timing and abandonment of a discrete investment project in the more general framework. He argues that hysteresis is caused by three features, namely irreversibility, uncertainty and the capability of waiting. As long as the opportunity to invest remains available and time brings more information about the future prospects of the project, a later decision can be a better one. However, the real options are not always beneficial to firms, because hysteresis reduces the corporate commitment to a planned project and possibly produces huge operating losses due to delayed abandonment decision. In other words, hysteresis encourages disinvestments for an idle firm and encourages investment for an active firm.

Stage development. Many investments occur in stages that must be carried out in sequence, and sometimes the payoffs from or costs of completing each stage are uncertain, and in addition they can be temporary or permanently abandoned if the value of the end product falls, or the expected cost of completing the investment rises. It could be the case, for example, of R&D projects. These can be viewed as a compound option; each stage completed gives the firm an option to complete the next stage. The problem is to find a contingent plan for making these sequential and irreversible expenditures. Majd and Pindyck (1987) solve this problem for a model in which a firm invests continuously. Often, however, sequential investments occur in discrete stages. In these cases, the optimal investment rules can be found by working backward from the completed project (see, e.g. Pindyck, 1991).

Growth options. So far we have examined decisions to invest in single, discrete projects. Another part of the literature focus on the incremental investment. Bertola (1989) and Pindyck (1988) developed models of incremental investment and capacity choice that account for irreversibility. In Pindyck's model the firm faces a linear demand function and has a Leontief technology. The firm can invest at any time at a cost k per unit of capital, and each unit of capital gives it the capacity to produce up to one unit of output per period. At the optimum, the present value of the expected cash flow from a marginal unit of capacity just equals the total cost of that unit, i.e., the purchase cost and the installation cost plus the opportunity cost of exercising the option to buy one unit. The investment problem is solved by first determining the value of an incremental unit of capital given the shock and an existing level of capital stock, K, and then finding the value of the option to invest in this unit and the optimal exercise rule. These operating options are worth more, the more volatile is demand, just as a call option on a stock is worth more, the more volatile is the price of the stock. This suggests that the firm should hold more capacity when future demand is uncertain, but the opposite is true. The reason is that uncertainty also increases the value of the firm's investment operations, and hence the opportunity cost of irreversible investment. Although the value of a unit capacity increases, this opportunity cost increases even more, so the net effect is to reduce the firm's optimal capacity. Pindyck shows that a more volatile demand implies that a firm should hold less capital, but have a higher market value. Dixit and Pindyck (1998) extend this model allowing for partial reversibility, which reflects the obsolescence of capital, and a costly capacity expansion, justified on the ground that continued entry or expansion by other firms pushes up capital costs. Their main result is to show how the investment and disinvestment decisions become separated when the initial gap between the purchase and resale price of capital is large.

Technology as growth option. Grenadier and Weiss (1997) recognize the impact that technological progress may have on investment decisions and study the strategies followed by firms in adopting a current technology. The risk the firm faces is that a new and a more efficient technology becomes available on the market. The evidence about the strategies followed is not unique. Some adopt new technologies when they are first available on the market, while others postpone the adoption decision until the technology is improved. Some firms will adopt every technological improvement, while others bypass

the innovation altogether. Grenadier and Weiss consider the innovation investment strategy as a sequence of embedded options [14]. Think, for example, at a firm contemplating investment in the early stage of a new technology. Not only is this firm investing in the current innovation itself, but it is also purchasing an option to upgrade in the future. As firms choose whether or not to adopt a current version of the innovation, they consider the implication for their ability to respond to future technological innovation. They also assume that there is uncertainty about both the value and the timing of the future innovations. These are a particular form of *compound option*. Under this conditions they derive the optimal "migration strategy" for adopting successive version of an innovation, the probability that a firm will pursue each of the identified migration strategies, as well as the expected time at which a firm will invest in innovation.

2.2.2. Irreversible Investment and Uncertainty

Economic theories do not succeed in predicting a clear-cut relationship between investment and uncertainty [15]. An early attempt to model investment as an irreversible process can be found in Arrow (1968), and this, together with the subsequent work in the same vein by Nickell (1978) demonstrates the importance of expectations in such a context. The role of uncertainty is implicit in the early adjustment costs literature in which the role of backward-looking expectations formation is captured through the inclusion of lagged variables. Recent authors, contributing to the growing literature on irreversible investments, criticize the old adjustment cost functions. The role of future expectations was made explicit in q-models of investment (Tobin, 1969) which rely on the assumption that current corporate stock valuation reflects agents' expectations about future conditions as they affect the firm in question. A key result, due to Hartman (1972) and extended in

[14] Mayers (1977) and Kester (1984) view a current innovation as a link in a chain of future investment options.

[15] See, e.g., Leahy and Whited (1996) to highlight some stylized fact about the effect of uncertainty on investment.

Abel (1983), was the finding that increased uncertainty can rise the marginal profitability of capital, and hence increases investment. In contrast, other authors have argued that irreversibility may lead to the postponement of the investment decisions (Bernanke, 1983; McDonald and Siegel, 1986). The real option literature emphasizes the role played by the degree of irreversibility and the option value of investment opportunities. All these models introduce uncertainty in the investment decision rule through a threshold value that triggers investment and show that uncertainty increases the investment trigger. In order to test this proposition, Guiso and Parigi (1999) try to proxy the firm's perception of risk by using the results of a survey study in which Italian firms are interviewed about their subjective ideas on the variability of future demand for their products. One of their main results is that the effect of uncertainty strongly differs across groups of firms due to differences in the reversibility of investment decision. In particular, it is stronger for firms that cannot easily reverse investment decision and for those with a substantial market power. These results are not without controversy. Caballero (1991) shows that the effect of an increase in uncertainty on irreversible investment is always positive if firm is operating in a perfectly competitive product market using a constant return to scale production technology. However, Pindyck (1993) argues persuasively that Caballero's result depends on treating the firm in isolation and ignoring industry-wide demand uncertainty. He shows that industry-wide uncertainty can have a negative effect on irreversible investment even under perfect competition and constant return to scale. Bal-Ilan and Strange (1999) point out that uncertainty may have a positive effect on irreversible investment. This might be the case when the intensity of investment is taken into account. However, these studies, focusing on the irreversibility of investment expenditures, do not consider several other channels through which uncertainty might affect investment decisions. First, we should consider the risk attitude of the firm. Nakamura (1999) theoretically proves that the degree of relative risk aversion alters the effect of uncertainty on investment. Introducing uncertainty and an expected utility maximization, he derives the behaviour of a risk averse firm and the conditions on the degree of risk aversion to get a negative investment-uncertainty relationship using a Cobb-Douglas production function. Aizenman and Marion (1999), in an empirical cross country study, pay attention to risk

averse behaviour. By modifying the standard expected utility analysis with the concepts of disappointment aversion (Gul, 1991) and Knightian uncertainty, they show that the effect of uncertainty on investment is negative. In general, one can conclude that the probability of a negative effect of uncertainty on investment increases the more risk-averse the firm is. Second, these models consider a linear investment-uncertainty relationship. However, some theoretical studies suggest that the investment-uncertainty relationship is a U-shaped curve. The reason is that individuals are risk seeking over the domain of small losses if the utility of an agent depends on gains and losses but not on the final income states (Kahaneman and Tversky, 1979). Sarkar (2000), considering the investment decisions in a real option fashion, points out two effects of uncertainty on investment. On the one hand, in line with the real option theory, uncertainty increases the investment threshold and therefore reduces the investment. On the other hand, computing the probability of investing, i.e. the probability of reaching the critical threshold, he shows that uncertainty increases the probability that the investment threshold will be hit. Therefore, higher level of uncertainty might have a positive effect on investment. This finding may suggest a U-shaped investment-uncertainty relationship. Cappuccio and Moretto (2001) highlight the role played by two crucial assumptions of Sarkar's model, namely the perpetual investment opportunity and the fact that the CAPM holds despite the real option approach does not rely on this assumption. Relaxing these two assumptions they show how uncertainty may have a controversial effect on investment. Bo and Lensink (2001) confirm the non-linear relationship hypothesis by providing evidence for a panel of Dutch manufacturing firms. Low levels of uncertainty have a positive effect on investments, while high levels of uncertainty lower investments. Finally, considering all-equity financed investments, these models do not consider the degree of financial constraints a firm faces in undertaking an investment. Ghosal and Loungani (2000) document that the investment-uncertainty relationship depends on the degree of capital market imperfections. A well developed financial system probably manages risks more efficiently. Moreover, in countries with well developed financial markets it is much easier to sell used goods, and hence the degree of irreversibility will probably be lower. Therefore, a firm facing an increase in uncertainty will more likely cut investments in the case where it suffers from financial constraints. Carruth et al. (2000) reviewed the recent developments in modelling investments under uncertainty in presence of irreversibility and have surveyed the growing literature that has endeavoured to examine empirically the insights that this new theory has provided. We do not explore the huge empirical literature and refer the reader interested in the empirical methodology to the excellent survey of Carruth et al. (2000). We limit ourselves to point out that a negative relationship between investment and uncertainty seems to be confirmed.

3. Strategic Interactions and Real Options

The models surveyed so far are based on two extreme market structures, namely monopoly, i.e. the investment opportunity is exclusive to the firm only, and perfect competition, i.e. the firm is a price taker. Let us now analyze the intermediate case of imperfect competition, and focus on the interaction between uncertainty and irreversibility in a strategic setting. Real options, in fact, could make a significant difference in the area of strategy and competition. Very little has been done on this topic and it is a promising avenue for future research. Literature regarding the option of waiting suggests that, in absence of opportunity costs or dividend-like effects due to waiting, a firm should wait until related information is favourable to exercise the option before expiration. However, in reality, a firm may find it desiderable to exercise the option to invest at an early stage at expense of scarifying the value of flexibility. This is because the early strategic investments may generate a certain value of commitment depending on its strategic impact. In what follows, we present a continuous time model, and we then analyze some extensions and applications of this model.

3.1. *Strategic Interactions: a Simple Continuous Time Model*

Smets ([16]) (1991) develops a stochastic game model in continuous

([16]) This treatment follows Dixit & Pindyck (1996, chapter 9). In the original framework, Smets analyses the decision of a firm on when to switch from exporting to foreign

time in which two firms are competing to enter a market paying a sunk cost I. The two firms can produce a unit of output flow with no variable cost of production. The industry demand, $D(Q)$, is sufficiently elastic to ensure full capacity production and industry output is 0, 1 or 2 depending on the number of active firms. The price is given by

$$P = Y(t)\, D(Q) \tag{13}$$

and the multiplicative shock $Y(t)$ follows a geometric Brownian motion. Firms are assumed to be risk neutral. Smets (1991) builds his solution using the framework developed by Dutta and Rustichini (1991). Assuming that one firm has already invested, they find the optimal decision of the other (the follower). Then, they consider the case where no firm has invested, and find the optimal decision of either as it contemplates whether to go first, knowing that the other will react in the way just calculated.

The follower's instantaneous profit flow will be $Y(t)\, D(2)$. Solving the usual stochastic optimal stopping problem it is possible to compute the threshold Y_2 that triggers the investment and the follower's value:

$$V_2(Y) = \begin{cases} \left(\frac{Y(t)}{Y_2}\right)^{\beta_1} \left[Y_2 \frac{D(2)}{r-\alpha} - I\right] & Y(t) < Y_2 \\ Y(t)\, D(2) - I & Y(t) \geq Y_2 \,. \end{cases} \tag{14}$$

The next step is to evaluate the value of becoming a leader, $V_1(Y)$, i.e., the value of being the first to invest. In making this calcula-

direct investment (FDI) to service a stochastically growing market. He assumes that producing at home requires a higher variable cost than producing abroad because of higher cost of labour. Building a foreign plant requires, however, a fixed and irreversible investment cost. As a result, FDI only takes place when foreign demand is large enough to cover the fixed investment costs. He characterises two equilibria. In the first equilibrium both firms enter the foreign market jointly. The second equilibrium is characterised by preemptive and sequential FDI.

tion, this firm will take into account the action of the other firm after it observes that its rival has already invested. If $Y(t) < Y_2$, then the follower will wait until Y_2 is hit. In the meantime, the leader will have the larger monopolistic profit flow $Y(t)\,D(1)$ and its expected value will be

$$V_1(Y) = E\left[\int_0^T e^{-rt} Y(t)\, D(1)\, dt\right] + E\left[e^{-rT}\right] Y_2 \frac{D(2)}{r-\alpha} - 1 \qquad \text{if } Y < Y_2 \tag{15}$$

where, as before, T is the first time the stochastic process of the demand shock reaches Y_2 starting at Y. If $Y(t) \geq Y_2$ the follower will invest at once, the leader's instantaneous profit flow will also be $Y(t)\,D(2)$, and its value will be the same as the follower's given in equation (14). The leader's value is

(16)

$$V_1(Y) = \begin{cases} \left(\frac{1}{r-\alpha}\right) Y\, D(1) \left[1 - \left(\frac{Y}{Y_2}\right)^{\beta_1 - 1}\right] + \left(\frac{Y}{Y_2}\right)^{\beta_1} \left[Y_2 \frac{D(2)}{r-\alpha}\right] - I & \text{if } Y < Y_2 \\ V_2(Y) & \text{if } Y \geq Y_2 \end{cases}$$

Having derived $V_1(Y)$ and $V_2(Y)$, it is possible to derive the equilibria of the duopolistic timing game. For a range of $Y < Y_2$ the leader's value exceeds that of the follower because it enjoys a higher profit flow before the follower invests. However, for a range of very low values of Y, the leader's value is less than the follower's because the leader incurs the investment cost up front but has a low profit flow initially. The two curves cross at the point Y_1. If the initial Y is below Y_1, neither firm will invest. When Y_1 is reached, one firm will invest at once and the other will wait until Y_2 is reached, but the two are indifferent between the two roles (note that $V_1(Y_1) = V_2(Y_1)$). Although the value of waiting is zero, the firm contemplating being the first to invest recognizes that the future entry by the other firm will reduce the upper end of the distribution of profits flows. Therefore, it requires a premium in compensation. The general point is not difficult to state. On the one hand, uncertainty and irreversibility imply an option value of waiting, and therefore greater hesitancy in each firm's investment decision. The fear of pre-emption by a rival, on the other hand, suggests the need to act quickly. Which of these considerations is the most important depends on the parameters of the problem, and on the current state of the underlying shock. How can this framework be used to explain economic phenomena?

3.2. *Technology and Competitions*

The model presented in the previous subsection is fairly simple, but illustrates how competition can affect an investment opportunity and its option value of waiting. These insights have been extended to a variety of problems involving investments decisions. A critical component of many firms' investment policies is the strategy for the adoption of technological innovation. In their seminal paper, Fudenberg and Tirole (1985) study a scenario of a duopoly with identical firms that both have the option to upgrade their technology. To do so, they have to pay a sunk cost that decreases over time. Thus, the later a firm acquires this technology the less it costs. It follows that the optimal investment decision faces a trade off in the sense that investing soon implies that the firm can produce more efficiently from an early point in time onwards, but on the other hand large sunk costs have to be paid. They consider endogenous firm's roles so that it is not determined beforehand which firm will be the first investor. They show that the outcome is either a pre-emption equilibrium with dispersed investment timings and rent equalization, or a joint adoption equilibrium.

Recognizing the importance played by technological progress in determining competition among firms, the Fudenberg and Tirole framework has been combined with the theory of investments under uncertainty, assuming that firm's profit flow satisfies a geometric Brownian motion. Huisman and Kort (1999) analyze the case in which a new technology is already available, and two active firms compete *à la* Cournot on the output market. They determine the optimal strategy of adoption, given that the new technology lowers variable production costs and confers a competitive advantage to the firm on the output market. Huisman and Kort (2000) develop further this concept considering the possibility of uncertainty over the arrival of innovations, and study the optimal adoption strategy according to the speed of innovation.

Huisman and Kort (1999) extend the Smets (1991) model described in Dixit and Pindyck (1996, Chapter 9). They allow that before the moment of investment the firms are already active on the output market on which they compete. In particular, they analyze the case in which two firms have the possibility to adopt a new technology paying a sunk cost I which after adoption increases the firm's profit. By how much this pay off is raised is not known beforehand, since the future market conditions for the firm's products are uncertain. After identifying the value to be the leader and the follower they compute the possible equilibria of the game. They identify three different scenarios. In the first scenario a pre-emption equilibrium occurs, where the moments of investment of both firms are dispersed. In the second scenario the outcome is that firms simultaneously invest when demand is relatively large. They call this a collusive equilibrium. In the third scenario it turns out that in economic environments with low uncertainty the pre-emption equilibrium is applied, while with large uncertainty both firms invest together at the moment that demand is large. Their main result is that in the pre-emption equilibrium, situations occur where it is optimal for one firm to invest, but at the same time investment is not beneficial if both firms decide to do so. Nevertheless, since the firms are identical there is a possibility that still both firms invest at the same time, which leads to a low pay off for both of them. They obtain that such a coordination failure can occur with positive probability at the moments of time where the leader's payoff is strictly larger than the follower's payoff, contrary to the standard results in Dixit and Pindyck, Grenadier and Weeds (17).

Huisman and Kort (2000) focus on the strategic effects of the adoption of new technologies. A firm that invests today faces the risk that a much better technology becomes available at an unknown future time. This could be seen as an incentive to delay the investment. The

(17) Grenadier (1996) and Weeds (2000) make and ad hoc assumption with the aim to be able to ignore the possibility of simultaneous investment at points of time that this is not optimal. Grenadier (1996) in the context of real estate development assumes that "if each tries to build first, one will randomly (i.e. through the toss a coin) win the race".

possible invention of a more efficient technology raises the option value of waiting to invest in the current technology, but on the other hand the presence of a competitor may induce the firm to invest quickly, and thus forget about future technological progress. These new considerations could turn a pre-emption game into a war of attrition. This could happen when the first mover invests in the current technology while the second mover waits for the new technology to arrive and invests then in it. Compared to the strategy of its competitor, the benefits of the first investor are the monopoly profits gained during the period that starts at the moment of investment by the first investor and lasts until the moment that the second mover invests. However, these monopoly profits can be more then offset by the efficiency gain the second investor enjoys due to producing with a more efficient technology, which takes place after both firms have invested. Which is the effect of uncertainty in this model? In general higher uncertainty increases the threshold level and this implies that investment in the current technology will be delayed. Therefore, the probability that the new technology arrives before the investment is undertaken increases. Hence, the conclusion is that increased revenue uncertainty induces higher probability that the new technology will be adopted. Moreover in a fast growing market, the firm is more willing to wait for a new technology, while an higher discount rate induces the firm to invest in the current technology rather than waiting for the new one. Pawlina and Kort (2001) develop further the above analysis introducing the possibility of investment costs asymmetry. This is justified on the ground that firms have different access to capital markets. In such a case, the cost of capital of a liquidity constrained firm is higher than its counterpart with access to a credit line or with substantial cash reserves. Moreover, costs asymmetry may be due both to a different degree of organizational flexibility in implementing a new production technology or to different regulations. They show that when asymmetry is small, firms invest jointly. When the first-mover advantage is significant, the lower cost firm pre-empts the higher-cost firm, and, finally, when the asymmetry between firms becomes sufficiently large, firms exercise their investment options sequentially and their mutual decisions do not affect each other directly. Finally, to analyze welfare implications they develop an example where the investment increases product quality. They show that social welfare is maximized when none of the firms suffers from competitive disadvantage. However, if the investment cost is low an increase of the consumer surplus resulting from the early investment in the pre-emption equilibrium exceeds the loss of the firm's joint value associated with such an investment. This allows to conclude that an equal access to a new technology or market segment may not be socially optimal. Mason and Weeds (2000) analyze the case of irreversible adoption of a technology whose returns are uncertain when there is an advantage to being the first adopter but a network externalities to adopting when others also adopt. Using a microfoundation *a la Hotelling* (linear city model), they show that a small amount of uncertainty can cause the first mover to adopt the technology earlier than predicted by standard real option theory.

4. Applications and Extensions

Another important strand of the literature on real option focus on R&D. Corporate R&D management is characterized by its nature of uncertainty and strategic positioning. During the last two decades, the application of option pricing formula to R&D has become of interest and numerous studies have attempted to address how the real options analysis can help to draw the proper line between knowledge building and strategic positioning. The introduction of the option pricing theory to the practical R&D area has been stimulated by Copeland and Weiner (1990). Based on their consulting experience, Copeland and Weiner report that many R&D managers have attempted to break down R&D programs into a sequence of review points, where they can decide either to abandon the line of research or to provide additional funding for the next stage. Hamilton and Mitchell (1990) also report that due to the dissatisfaction with the return on investment analysis, the senior management at General Electric Co. incorporated the option thinking into the evaluation of the R&D programs. They argue that an important part of what R&D does is to create long term strategic options for the firm, and thus all R&D programs should be evaluated on the basis of such efforts. Although there is sufficient evidence showing management's awareness of the option thinking in the evaluation and selection

of R&D projects, researchers have been argued which theoretical real option model is more appropriate in valuing R&D projects. For example, Pennings and Lint (1997, 1998) argue that the widely accepted assumption of Brownian Motion is inappropriate in modelling the NPV of the new product at the moment of industrialization due to discontinuous unforeseen events. Geometric Brownian motion implies a continuous arrival of information which changes the value of the underlying asset. However, in the R&D area, management will re-evaluate the research project only if information with strategic impact arrive. Consequently they suggest the discontinuous jump process to describe the nature of R&D investments, in which the arrival of information affecting changes in the project value is Poisson-distributed. They than analyze the R&D investments as a compound options. Focusing more on the strategic aspect of R&D investments, Weeds (1999) develops a model of irreversible investment in which two identical firms have the opportunity to invest in competing research projects. The first to invent gains monopoly profits in the relevant product market. She assumes both technological and economic uncertainty. Discovery occurs randomly, while the value of the patent received by the successful inventor evolves accordingly to a stochastic process. Thus the model is a stochastic stopping timing game in which each firm's optimal trigger strategy is given by a critical value for the underlying stochastic process such that the firm invests at the first time that this point is reached. Weeds shows that firms will delay investment to a greater extent than in the absence of competition. When one firm invests in research its rival's option to delay is reduced in value since there is some probability that the discovery will be made before this firm invests. In a non-cooperative framework each firm ignores the destruction of its rival's option value. Taken on its own this business-stealing effect would induce earlier investment. However, forward looking firms take account of their rivals reactions in formulating their optimal strategies. Over a range of patent values investment by one firm induces its rival to invest at once, resulting in a surge of research activity and a race for patent. Anticipating this reaction a firm may choose to delay investment to a greater extent than in the absence of competition. In effect, an investing firm chooses the time at which the patent race will begin and it is better for both firms if it is delayed until the jointly optimal investment point is reached.

The real option valuation approach has recently been applied to the real estate development and provides a potential explanation for several puzzling real estate market activity.

The fact that investors choose to keep their valuable land vacant or underdeveloped for prolonged periods of time as a potential site in future suggests that the vacant land is more flexible and valuable to investors while they "keep the option" open. Titman (1985) adopts the same methodology suggested by Black and Scholes (1973) and Merton (1973) to value vacant urban land as an option to choose different types of building now or in the next period. Titman argues that traditional valuation approach to land development which identifies the most possible future use of the land, appraises the expected cash flow due to the use, and then discounts the future value to the present, generally ignores flexibility contingent on the real estate prices. Thus, he suggests to treat the vacant land as a call option with a number of possible buildings as the underlying asset and their building costs as exercise price. Two important implications drawn from Titman's analysis need to be addressed. Firstly, the uncertainty related to the real estate prices has a positive effect on the vacant land. The more uncertain the price is, the more valuable the land is. Secondly, the analysis poses some propositions regarding government policy. For example, the initiation of the height restrictions on building for the purpose of limitating growth may lead to an immediate effect on increasing the building activity due to decrease of future uncertainty. The approach of viewing the vacant land as an option is limited by the perfect capital market assumption which may be less realistic for the real estate markets due to its non-homogeneousity nature. Quigg (1993) empirically examines the option to wait to develop a land. Her model is an extension of Williams' (1991) in which unit development cost and unit cash inflow to developed property are the underlying stochastic variables. She changes these variables to total construction cost and total building price for the purpose of empirical setting. The statistical results find a mean option premium of 6% of the theoretical land value, in which the "option premium" is defined as the difference between the intrinsic value and the option model value, divided by the option model value. Capozza and Li (1994) apply the optimal stopping approach in McDonald and Siegel (1986) to analyze the land use decisions with the inclusion of capital intensity. In the investment decisions regarding land use, variable intensity of capital has a fundamental effect since the projects are mostly site-specific. Their model shows that the ability to develop at a variable intensity raises the hurdle rent and delays development. Thus, developed property value contains not only the irreversibility premium as in other models, but also the intensity premium.

Grenadier (1996) developed a stochastic duopolistic game based on Smets (1991) highlighting the main features of the real estate market, namely the fact that the rent gained by a landlord is a decreasing function of the construction of new and superior building. He tries, in this way, to explain the irrational overbuilding phenomena.

5. The Financing Side

Since the early 80's, advances in the real options literature have completely changed the way we evaluate the investment opportunities. As shown in this literature, firms should not invest in projects which are expected to earn only the opportunity cost of capital. Managers can make choices about the project's characteristics and this flexibility creates embedded options. These options add value to the project and invalidate the traditional NPV rule ([15]). Although this literature has made a great step toward a better understanding of investment decisions, little research has focused on the practical side of the investment spending. In fact, this literature typically assumes all-equity financed investment, or, alternatively, it is assumed that the Modigliani-Miller theorem holds, so the firm's real investment decisions are independent of its financial structure, and therefore ignore the possible interaction among investment and financing decisions. This topic is first addressed by Mayers (1977) who showed how investment decision, i.e. acceptance or rejection of projects, affected the optimal financial structure of a firm and why investment should in turn be

([15]) Among them, we can quote the option to defer the investment spending (McDonald and Siegel (1986)), the option to abandon on active project (Majd and Myers (1990)) the option to expand or to reduce the production capacity (Abel and Eberly (1996)).

affected by leverage. In particular, he suggested that a firm's existing debt load causes underinvestment. Theoretical models examining the investment-financing linkage have almost exclusively been static, i.e. investment and financing decisions are made at a single point in time and are irreversible. Furthermore, most of these models have only endogenised either the investment or the financing decision. Recently, these topics, and more generally the link between the optimal capital structure and the operating decisions, have drown the attention to some articles that apply the real option approach. Mauer and Triantis (1994) analyze the interaction between a firm's optimal dynamic investment, operating and financing decisions in a contingent claim model. They consider a monopolistic firm that produces a single commodity whose price follows a geometric Brownian motion. The firm can shutdown and reopen its production facility in response to price fluctuation by paying operating and adjustment cost. To finance production the firm may issue both debt and equity and can alter its capital structure over time by paying a recapitalization cost. By varying operating adjustment and recapitalization costs, they explore the impact of production and financial flexibility on the interaction between the firm's investment, operating and financing decisions in a firm value market framework. From numerical simulations, they find that production flexibility has a positive effect on the value of the interest tax shields. This is because the ability to shut down operations allows the firm to mitigate operating losses, reducing the variance of the firm value, and therefore increasing debt capacity and the associated net benefit of interest tax shields. They also find that debt financing has a negligible impact on the firm's investment and operating profits. While a levered firm has incentive to invest earlier than an equivalent unlevered firm because it earns interest tax shields when it is operating, the benefits from doing so is largely offset by a loss in the value of waiting to invest. Therefore, the net benefit is not large enough to effect a significant change in investment policy. Thus a levered firm can determining the exercise timing decision on its real options, ignoring the effect of debt financing. Freis, Miller and Perraudin (1997) study the other polar case of firms' entry and exit decisions under perfect competition and their implications for the pricing of corporate debt. Their model is close to Dixit's (1989) but they allow for debt financing. They show that free entry and exit impose reflecting barriers on the price of the

industry's output, affecting the valuation of corporate debt and equity and the optimal capital structure decision. Mauer and Ott (2000) suggest that a firm which faces greater uncertainty will expand debt capacity by waiting for a better state to exercise the investment option. The only model which proposes a unifying framework for analyzing the interaction between investment and financing decision in a duopolistic market is Lambrecht (2001). The author presents a dynamic contingent claim model of strategic entry and exit in a duopoly. Introducing debt financing in his model, he analyses the order in which firms go bankrupt within a given industry, and the extent to which this order is influenced by aggregate economic factors. In case of market exit, they find that higher operating profits, lower debt repayments, larger incremental gains from becoming a monopolist and higher bankruptcy costs give a firm a competitive advantage versus its competitors in the battle for survival. Changes in the general economic environment can therefore reverse the order in which firms are expected to foreclose. Focusing on the market entry, they find that the follower's need to borrow money tends to delay its entry, whereas any financial vulnerability of the incumbent firm tends to speed up entry. The latter finding support those articles which argue that leverage makes product-market competition softer.

6. Conclusions

In recent years there has been a considerable interest in real options as a valuation method and investment appraisal technique, now applied to a number of areas including valuation of R&D, patents and real estate. As we have seen through this paper, the real options techniques are more complex than the established investment appraisal methods of discounted cash flow (DCF) or return on capital employed (ROCE). Natural questions we have tried to answer in this work are: when exactly should the real options be applied? What does this approach imply? How can we identify situations in which real options should be abandoned in favor of more established techniques?

Real options valuation can be used when all of the following conditions apply: i) the future is uncertain; ii) the investment decision is irreversible, whether fully or in part; iii) the firm holding the (investment) option has the ability to delay.

Under these three conditions there is an option value to delaying investment. This option value should be added to the cost of making an investment – it is an opportunity cost that is incurred when investment takes place. Since options values are positive, real options raise the hurdle for investment: investment should not take place until the expected profit exceeds the investment cost weighted by the option value. Thus, real options analysis usually implies that investment should be delayed, compared with traditional DCF analysis.

As the real options literature is still growing rapidly, empirical evidence on the explanatory power of real options models started quite recently. The first interesting direction for further research is to build an empirical database for testing validity of real options models throughout a variety of applications. Comparing and testing real options models with conventional alternative approaches of valuation can provide a profound insight on consistency and validity of real options models. As mentioned previously, real options problems have been explored in the context of strategy and competition, yet its application under a variety of competitive and market structure is relatively limited. Particularly in the case of new product development or market test with major competitors, it is important to incorporate competitive counteractions into the analytical framework. Trigeorgis (1999) and Howell et al. (2001) suggest to explore in more depth endogenous strategy and competition using a combination of the option approach and game theory.

Acknowledgement

We would like to thank Giuseppe Colangelo, Giovanni Marseguerra and PierCarlo Nicola for many helpful discussions and suggestions. The usual disclaimer applies. This paper has been realised within the research program "Modelli non lineari in economia e finanza: dinamiche complesse, disequilibrio, integrazioni strategiche". Financial support from Ministero dell'Università e della Ricerca Scientifica e Tecnologia - COFIN-MIUR 2002 is gratefully acknowledged.

R E F E R E N C E S

[1] Abel A.B., 1983, *Optimal investment under uncertainty*, American Economic Review n. 73, pp. 228-233.

[2] Akerlof G.A., 1970, *The Market for Lemons: Quality Uncertainty and Market Me chanism*, Quarterly Journal of Economics, n. 84, pp. 448-500.

[3] Arrow K.J., 1968, *Optimal Capital Policy with Irreversible Investment*, in *Valu capital and growth, papers in honor of Sir John Hicks*. Ed.: J. Wolfe. Edinburgh Edinburgh University.

[4] Aizenman J. and N.P. Marion, 1999, *Volatility and investment: interpreting evidenc from developing countries*, Economica, n. 66, pp. 157-179.

[5] Bar-Ilan, A. and W.C. Strange, 1999, *The timing and intensity of investment*, Journa of Macroeconomics, n. 21 (1), pp. 57-77.

[6] Bertola G., 1989, *Irreversible Investment*, Mimeo, Princeton University.

[7] Bo H. and R. Lensink, 2001, *Is the investment-uncertainty relationship non-linear An empirical analysis for the Netherlands*, Mimeo.

[8] Brennan M.J. and E.S. Schwartz, 1985, *Evaluating Natural Resource Investment* Journal of Business, n. 58, January, pp. 135-157.

[9] Brennan M.J. and L. Trigeorgis (eds.), 2000, *Project, flexibility, agency, and compet tion: New developments in the theory and application of real options*, Oxford, N Oxford University Press.

[10] Caballero, R. J., 1991, *Competition and the nonrobustness of the investment-unce tainty relationship*, American Economic Review, n. 81 (1), pp. 279-288.

[11] Capozza D.R. and L. Yuming, 2001, *Residential Investment and Interest Rates: a Empirical Test of Land Development as a Real Option*, Real Estate Economic n. 29, pp. 503-519.

[12] Cappuccio N. and M. Moretto, 2001, *Comments on the Investment-Uncertainty Rel tionship in a Real Option Model*, Nota di Lavoro n. 28.2001, Fondazione Eni Enric Mattei.

[13] Carruth A., A. Dickerson and A. Henley, 2000, *What do we know about investme under uncertainty?*, Journal of Economics Surveys, n. 14, pp. 119-153.

[14] Copeland T. and J. Weiner, 1990, *Proactive Management of Uncertainty*, McKins Quarterly, n. 4, pp. 133-152.

[15] Dixit A., 1989a, *Entry and Exit Decisions under Uncertainty*, Journal of Politic Economy, Vol. XCVII, n. 3, pp. 620-638.

[16] Dixit A., 1991, *Irreversible Investment with Price Ceilings*, Journal of Political Economy, Vol. XCIX, June, pp. 541-557.

[17] Dixit A., 1993, *The Art of Smooth Pasting*, Vol. 55 in *Fundamentals of Pure and Applied Economics*, eds. Jacques Lesourne and Hugo Sonnenschein, Chur, Switzerland, Harwood Academic Publishers.

[18] Dixit A. and R.S. Pindyck, 1994, *Investment under Uncertainty*. Princeton, Princeton University Press.

[19] Dixit A. and R.S. Pindyck, 1998, *Expandability, Reversibility, and Optimal Capacity Choice*, NBER Working Paper N. W6373.

[20] Dixit, A., R.S. Pindyck and S. Sødal, 1997, *A Mark up Interpretation of Optimal Rules for Irreversible Investment*, NBER Working Paper N. 5971.

[21] Dutta P.K. and A. Rustichini, 1991, *A Theory of Stopping Time Game with Applications to Product Innovation and Asset Sales*, Discussion Paper N. 523, Department of Economics, Columbia University.

[22] Freis, M. Miller Perraudin, 1997, *Debt in Industry Equilibrium* , Review of Financial Studies, n. 10 (1), pp.

[23] Fudenberg D. and J. Tirole, 1985, *Preemption and Rent Equalization in the Adoption of New Technology*, Review of Economic Studies, Vol. LII, pp. 383-401.

[24] Ghosal V. and P. Loungani, 2000, *The differential impact of uncertainty on investment in small and large businesses*, The Review of Economics and Statistics, (May), n. 82 (2), pp. 338-349.

[25] Grenadier S.R., 1996, *The Strategic Exercise of Options: Development Cascades and Overbuilding in Real Estate Markets*, The Journal of Finance, (December), n. 5, pp. 1653-1679.

[26] Grenadier S.R. and A.M. Weiss, 1997, *Investment in technological innovation: An option pricing approach*, Journal of Financial Economics, n. 44, pp. 397-416.

[27] Guiso L. e G. Parigi, 1999, *Investment And Demand Uncertainty*, The Quarterly Journal of Economics, vol. 114, issue 1, pp. 185-227.

[28] Hamilton W.F. e G.R. Mitchell, 1990. *What is your R&D worth?*, McKinsey Quarterly, 3, pp. 150-160.

[29] Huisman K.J. and P. Kort, 1999, *Effect of Strategic Interactions on the Option Value of Waiting*, Discussion Paper, Department of Econometrics and CentER, Tilburg University.

[30] Huisman K.J. and P. Kort, 2000, *Strategic Technology Adoption Taking Into Account Future Technological Improvements: A Real Option Approach*, Discussion Paper, Department of Econometrics and CentER, Tilburg University.

[31] Kahaneman D. and A. Tversky, 1979, *Prospect Theory: an Analysis of Decision under Risk*, Econometrica, n. 49, pp. 263-291.

[32] Kester W.C., 1984, *Today's Options for Tomorrow's Growth*, Harvard Business Review, March/April, pp. 153-160.

[33] Lambrech B.M., 2001, *The Impact of Debt Financing on Entry and Exit in a Duopoly* The Review of financial Studies, v. 14, n. 3.

[34] Leahy J.V. and T.M. Whited, 1996, *The effect of uncertainty on investment: some stylized facts*, Journal of Money, Credit, and Banking, n. 28 (1), pp. 64-83.

[35] Lint O., 2001, *Retrospective insights from Real Options in R&D*, Mimeo.

[36] Luehrman T.A., 1998, *Strategy as a Portfolio of Real Options*, Harvard Business Review, pp. 88-99.

[37] Majd S. and R.S. Pindyck, 1987, *Time to build, Option Value, and Investment Decisions*, Journal of Financial Economics, n. 18, pp. 7.

[38] Marzano F. e E. Saltari, 1999, *Teoria delle decisioni di investimento in condizioni di certezza e incertezza. Una rassegna critica*, in G. Zanetti (ed.), *Le decisioni di investimento*, il Mulino, Bologna.

[39] Mason R. and H. Weeds, 2000, *Networks, Options and Preemptions*, Working Paper, University of Southampton.

[40] Mauer, and Triantis, 1994, *Interaction of Corporate Finance and Investment Decision: a Dynamic Framework*, Journal of Finance, n. 49 (4), pp.

[41] Mauer D.C., and S.H. Ott, 2000, *Agency costs, underinvestment and optimal capital structure*, in *Project flexibility, agency, and competition: New developments in the theory and application of real options*. Ed. M. Brennan and L. Trigeorgis Oxford, NY: Oxford University Press.

[42] Myers S.C., 1977, *Determinants of Corporate Borrowing*, Journal of Financial Economics, n. 5, pp. 147-176.

[43] Myers S.C. e S. Majd, 1984, *Calculating Abandonment Value Using Option Pricing Theory*, Cambridge, MA, MIT Press.

[44] McDonald R. and D. Siegel, 1984, *The Value of Waiting to Invest*, Quarterly Journal of Economics, (November), n. 101, pp. 707-728.

[45] Metcalf and Hassset, 1995, *Investment under Alternative Return Assumptions Comparing Random Walks and Mean Reversion*, Journal of Economic Dynamics and Control, vol. 19, pp. 1471-1488.

[46] Moretto M., 1995, *Controllo ottimo stocastico, processi browniani regolati e optimal stopping*, Rivista Internazionale di Scienze Economiche e Commerciali, vol. 42, pp. 93-124.

[47] Nakamura T., 1999, *Risk aversion and the uncertainty-investment relationship: A note*, Journal of Economic Behavior and Organization, n. 38, pp. 357-363.

[48] Pawlina G. e P.M. Kort, 2001, *Real Options in an Asymmetric Duopoly: Who Benefits from Your Competitive Disadvantage*, CentER Discussion Paper N. 2001-95, Tilburg University, Tilburg, The Netherlands.

[49] Pennings E. e O. Lint, 1997, *The option value of advanced R&D*, European Journal of Operational Research, 103, issue 1, pp. 83-94.

[50] Pindyck R.S, 1988, *Irreversible Investment, Capacity Choice, and the Value of the Firm*, American Economic Review, 79, December, pp. 969-985.

[51] PINDYCK R.S, 1991, *Irreversibility and the explanation of investment behavior*, in LUND D. and OKSENDAL B. (eds.), *Stochastic Models and Option Values*, Amsterdam: North Holland.

[52] QUIGG L., 1993, *Empirical Testing of Real Option-Pricing Models*, Journal of Finance, n. 48, pp. 621-640.

[53] SARKAR S., 2000, *On the investment-uncertainty relationship in a real options model*, Journal of Economic Dynamics and Control, 24, pp. 219-225.

[54] SMETS F., 1991, *Exporting versus FDI: The Effect of Uncertainty, Irreversibilities and Strategic Interactions*, Working Paper, Yale University.

[55] TITMAN S., 1985, *Urban Land Prices under Uncertainty*, American Economic Review, n. 75, pp. 505-514.

[56] TRIGEORGIS L., 1996, *Real Options*, Cambridge, MA, MIT Press.

[57] WEEDS H., 1998, *Strategic Delay in a Real Options Model of R&D Competition*, Mimeo, University of Warwick.

[58] WEEDS H., 1999, *Sleeping Patents and Compulsory Licensing: an Options Analysis*, Warwick Economic Research, n. 577, University of Warwick.

[59] WILLIAMS J.T., 1991, *Real Estate Development as an Option*, Journal of Real Estate Finance and Economics, n. 4, pp. 191-208.