

Transversity K factors for Drell–Yan processes

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Abstract. The question of the K factor in transversely polarised Drell–Yan (DY) processes is examined. The transverse-spin case is peculiar for the absence of a reference point in deeply inelastic scattering (DIS). Therefore, in order to study more fully the possible effects of higher-order corrections on DY asymmetries, a DIS definition for transversity is devised using a hypothetical scalar (Higgs-like) vertex. The results show that some care may be required in interpreting experimentally extracted partonic transversity, in particular when comparing with model calculations or predictions.

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1 Introduction

The theoretical framework for describing transversity (at the basic level of partonic processes, QCD evolution, radiative effects, et cetera) is now solid [1] and a number of experiments aimed at its measurement are on-line or under development: HERMES [2], COMPASS [3] and the spin programme at RHIC [4]; there are also proposals for DY measurements with polarised antiprotons at the High Energy Storage Ring at GSI [5,6] (related preliminary theoretical studies have been made regarding accessing transversity in J/ψ production).

Transversity is the last remaining piece in the partonic jig-saw puzzle making up the hadronic picture. However, the standard procedure of using DIS as the process *defining* parton densities at the NLO cannot be extended to transversity in a simple manner since it does not contribute to DIS. Moreover, transverse-spin effects are notoriously surprising; see, e.g., the large and unexpected (historically) SSA [7,8]. These considerations render imperative the complete understanding of NLO corrections in DY before attempts are made to extract the partonic transversity distributions. See, e.g., [9] for a detailed discussion of transversity and also SSA.

One might also consider the double-spin asymmetry A_{TT} for other processes, such as $p^\uparrow p^\uparrow \rightarrow \text{jet} + X$, $\gamma + X$ et cetera. Unfortunately, however, A_{TT} is always very small [10,11], so that measuring transversity directly appears to be feasible only in doubly polarised $p\bar{p}$ interactions.

Since all QCD and EW vertices conserve quark chirality, transversity actually decouples from DIS. Chirality flip is not a problem though if the quark lines connect to different hadrons, e.g., as in the DY process. Unfortunately,

there is a caveat to accessing transversity in DY: Hikasa’s theorem [12], which states that, owing to chiral symmetry, transversity effects vanish upon integrating over the azimuth of the lepton pair. No simple proof of the theorem exists (it has to do with the γ -matrix algebra). Let us now make a few observations based on these properties of transversity.

- (1) The only “gold-plated” process in which transversity may be measured directly (i.e., without the need of more-or-less exotic fragmentation functions) is DY;
- (2) Hikasa’s theorem implies the use of a slightly less than fully inclusive process, in as much as one angle must be left unintegrated;
- (3) in the case of transversity asymmetries, helicity conservation may not necessarily provide the usual safeguard against large K factors.

These observations have non-trivial implications with respect to the measurement of transversity in DY and the interpretation of the results.

In the absence of a DIS reference point, there is no immediate way of evaluating the possible importance of the higher-order QCD corrections. The K factors are known to be large at the level of cross-sections in both the unpolarised [13] and helicity-dependent [14] cases. However, in the case of longitudinal polarisation the large corrections cancel in the asymmetry [14]. To a large extent this cancellation can be traced to the conservation of helicity along fermion lines in gauge theories – the $O(\alpha_s)$ Wilson coefficient for the DY process is identical for the helicity-dependent and -independent pieces (as too are the LO anomalous dimensions).

The coefficient function for the transversely polarised DY process differs significantly from the other two cases [15,16]. Moreover, the LO anomalous dimensions are dif-

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ferent – there is no corresponding conserved quantity or sum-rule.

Given the marked differences from the other two cases, it would seem useful to examine the question of DY K factors for transversity. In order to do this, it is clearly necessary to find some suitable DIS-like process as a reference point. The principal requirement is a spin-flip mechanism. A priori there are two obvious possibilities: a fermion mass term or a scalar vertex. Now, of course, DIS with transversely polarised leptons and nucleons should be considered and therefore the twist-three structure function g_2 (for general reviews, see, e.g., [17, 18]) is the natural object of study. It turns out, however, that although transversity is intimately related to the evolution of g_2 (the relevant operator is indeed proportional to the fermion mass [19–21]), at the level of direct contribution to polarised DIS it actually cancels against other higher-twist contributions owing to the equations of motion; see for example [22]. Although the calculation is rather delicate, the possibility of defining a coefficient function for transversity via its role in the evolution of g_2 has been examined [23], with similar results to those presented here.

A better and simpler approach is to identify some DIS-like process in which a scalar particle plays a role. Since the Higgs boson does indeed interact with quarks (as with leptons), the obvious solution to the problem is a gedanken process in which the exchange is no longer via the electroweak gauge fields but via the Higgs particle. To be precise, in order to obtain the required single spin-flip, Higgs–vector interference diagrams actually need to be considered. Of course, there is no intended suggestion here that such a process should really be measured, but merely that it forms a suitable basis for a theoretical cross-check. We should remark that such a process has effectively already been exploited for the calculation of $h_1(x)$ itself [24], on the basis of a suggestion by Jaffe. In any case, various tests will be performed to ensure that the results do not depend on the specific nature of the vertex introduced.

Of course, an alternative approach might be to work entirely in a DY-based scheme (i.e., for both the unpolarised and polarised analyses); this would, however, necessitate some reworking of most model predictions.

Thus, in the following section the calculations are described, the Higgs–vector interference mechanism is examined in detail and NLO calculation of the related Wilson coefficients is performed. The known results for the DY process are discussed and finally the relevant K factors are extracted. In the closing section some conclusions are drawn and comments relevant to future measurements of transversity via DY scattering are made.¹

2 The calculation

2.1 Drell–Yan cross-section and asymmetries

It is now standard to define the helicity- and transversity-dependent cross-sections by

$$\frac{d\Delta\sigma}{dQ^2} \equiv \frac{d\sigma^{++}}{dQ^2} - \frac{d\sigma^{+-}}{dQ^2}, \quad (1a)$$

and

$$\frac{d\Delta_T\sigma}{dQ^2} \equiv \frac{d\sigma^{\uparrow\uparrow}}{dQ^2} - \frac{d\sigma^{\uparrow\downarrow}}{dQ^2}, \quad (1b)$$

where the prefixes Δ and Δ_T indicate the longitudinal-spin (or helicity) and transverse-spin (or transversity) cases respectively, \pm refer to initial-state proton helicities and \uparrow, \downarrow to transverse polarisations. The double-spin asymmetries are then

$$A_{LL} \equiv \frac{d\Delta\sigma/dQ^2}{d\sigma/dQ^2}, \quad (2a)$$

and

$$A_{TT} \equiv \frac{d\Delta_T\sigma/dQ^2}{d\sigma/dQ^2}. \quad (2b)$$

The large NLO corrections afflict both the numerators and denominators. The question is to what extent they are correlated, i.e., to what extent they are the same and thus cancel in the ratio.

Turning then to the calculation of the K factor, the procedure will be essentially identical to that followed in earlier work [13, 14] and thus we shall not dwell on the general technicalities, save for those points that are significantly different in the case of transverse polarisation. The first peculiar aspect to be exploited is that, owing to the charge-conjugation properties of the relevant operator, the evolution of transversity is of the flavour NS type. In the NS case the effect of higher-order corrections may be represented in the following schematic way:

$$F(x, t) = \int_x^1 \frac{dy}{y} \sum_f Q_f^2 \times \left[\delta \left(1 - \frac{x}{y} \right) + \frac{\alpha_s(Q^2)}{2\pi} t P \left(\frac{x}{y} \right) + \frac{\alpha_s(Q^2)}{2\pi} C \left(\frac{x}{y} \right) \right] q_f(y, t), \quad (3)$$

where $t \equiv \ln(Q^2/\mu^2)$, with Q^2 the virtuality of the photon, $q_f(y, t)$ and Q_f are respectively the parton density and charge of quark flavour f , P is the universal quark–quark splitting function and C the process-dependent Wilson coefficient. The quantity $F(x, t)$ on the LHS then represents a generic (flavour NS) structure function and the three terms inside the square brackets on the RHS represent (1) the LO point-like contribution; (2) the LL correction; and

¹ Following correction of an error in the code used for the numerical estimates, the results shown here are a little less dramatic than those already presented by the author in some past conferences.

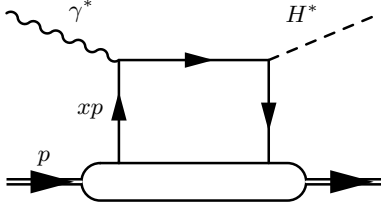


Fig. 1. The DIS “handbag” diagram for a photon–Higgs interference process

(3) the NLO correction. It is this last one that is of interest here.

To NLO the DY cross-section for $p\bar{p}$ scattering is expressed in term of parton densities as follows:

$$Q^2 \frac{d\sigma^{\text{DY}}}{dQ^2} = \frac{4\pi\alpha}{9s} \int_0^1 dx_1 dx_2 dz \delta(x_1 x_2 z - \tau) \times \sum_f Q_f^2 \left[q_f(x_1, Q^2) \bar{q}_f(x_2, Q^2) + (1 \leftrightarrow 2) \right] \times \left[\delta(1-z) + \frac{\alpha_s(Q^2)}{2\pi} C^{\text{DY}}(z) \right], \quad (4)$$

where $\tau = Q^2/s$, Q^2 is the invariant mass squared of the lepton pair and s is the total hadron CM energy squared. In (4) $x_{1,2}$ are the momentum fractions carried by the (anti)quarks inside hadrons 1 and 2 respectively. It is then the difference between the DIS corrections, with which the NLO parton distributions are defined via (3), and NLO DY corrections that constitutes the theoretical K factor.

The LL splitting functions P are well known [25–30] and may be expressed in the following compact form:

$$\Delta P(z) = P(z) = C_F \left[\frac{1+z^2}{1-z} \right]_+, \quad (5a)$$

$$\Delta_{\text{T}} P(z) = P(z) - C_F(1-z). \quad (5b)$$

The definition of the so-called “plus” regularisation is recalled in Appendix A. Already then it is evident that although fermion-helicity conservation guarantees identical evolution for NS spin-averaged and helicity-dependent quark densities, the same does not hold for the transversity case.

The problem now is to calculate the coefficient $C(z)$ of the third term in (3). This must be done for both the DIS and DY processes. As is well known, a large part of the DY K factor can be attributed to the change from the space-like Q^2 in DIS to time-like in DY. However, this is not the only origin of large effects and one should be concerned that the case of transversity with the extra requirement on the final-state phase space might introduce important differences.

2.2 The Drell–Yan process

Since the results for the DY process are known [15, 16], it is perhaps better to begin with this coefficient. The partonic

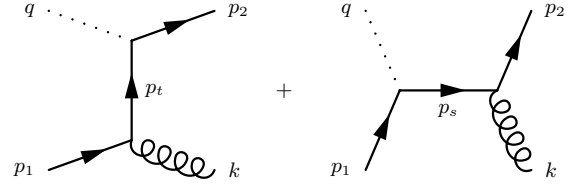


Fig. 2. The two diagram types contributing to the NLO DIS hard partonic $q\gamma^*(H^*) \rightarrow gq$ scattering subprocess, the dotted line represents either a virtual photon or Higgs

subprocess to be calculated is shown in Fig. 2. The virtual photon then decays into the final lepton pair of which (for the case of transversity) the azimuthal angle must be left unintegrated. The results for the DY unpolarised [13], helicity [14] and transversity [16] coefficient functions are as follows:²

$$C^{\text{DY}}(z) = C_{\text{L}}^{\text{DY}}(z) = C_F \left\{ 4(1+z^2) \left[\frac{\ln(1-z)}{1-z} \right]_+ - \frac{2(1+z^2)\ln z}{(1-z)} + \left[\frac{2}{3}\pi^2 - 8 \right] \delta(1-z) \right\}, \quad (6a)$$

$$C_{\text{T}}^{\text{DY}}(z) = C^{\text{DY}}(z) + C_F \left\{ -4(1-z)\ln(1-z) + 2(1-z)\ln z - \frac{6z\ln^2 z}{(1-z)} + 4(1-z) \right\}. \quad (6b)$$

It is important to note that although the large π^2 terms and indeed the entire coefficient of the δ function (which constitute the bulk of the K factor) appear invariant here, in the transversity case there is a new term, $\frac{6z\ln^2 z}{(1-z)}$, not found in the others.

2.3 Deeply inelastic scattering

In order to accommodate a spin-flip in the standard “handbag” diagram of DIS one of the vertices should involve a Higgs-like scalar; see Fig. 1. The contribution of this diagram can be expressed in terms of a structure that, for brevity, will be called h_1 here. Projecting with $\gamma_5 \not{p} \not{q}$ then leads to

$$W^\mu = h_1(x, Q^2) \frac{i\epsilon^{qps\tau\mu}}{p \cdot q}. \quad (7)$$

2.3.1 Real-gluon contributions

The NLO Wilson coefficient may now be calculated from the diagrams in Fig. 2. The process to be calculated is, of

² Here and in what follows only the unpolarised result will be presented in full while the helicity and transversity cases will be shown as differences with respect to the unpolarised case.

course, still photon–Higgs interference. The use of dimensional regularisation poses the problem of dealing with γ_5 , which naturally arises in the case of polarised DIS (for both helicity and transversity) owing to the projector $\gamma_5 \not{p} \not{s}_T$. Note that for the $q\bar{q} \rightarrow \gamma^* g$ DY subprocess this is not a problem since both the quark and antiquark bring on power of γ_5 , which then cancels before calculating any traces. The technique adopted here is that of defining a fully anti-commuting but non-cyclic γ_5 (see for example, [31]); for a detailed discussion of this technique, see also [32]. Consistency then requires that all traces be evaluated from the same reference point, which in the usual DIS case is unambiguously the one of the photon vertices. Here the scalar vertex could also be chosen – an explicit check shows that there is no ambiguity in the results.

In the MMS scheme (for a working definition, see Appendix B), adopting the above-mentioned γ_5 scheme and suppressing (for clarity) a common factor

$$C_F \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \quad (8)$$

on the RHS of all equations, the results for the real contributions of the diagrams in Fig. 2 are

$$\begin{aligned} \tilde{C}^{\text{DIS-R}}(z) &= \frac{2}{\epsilon^2} \delta(1-z) - \frac{1}{\epsilon} \left[\frac{1+z^2}{(1-z)_+} - \frac{3}{2} \delta(1-z) \right] \\ &+ (1+z^2) \left[\frac{\ln(1-z)}{1-z} \right]_+ - \frac{3}{2} \frac{1}{(1-z)_+} + 3 + 2z \\ &- (1+z^2) \frac{\ln z}{(1-z)} + \frac{7}{2} \delta(1-z), \end{aligned} \quad (9a)$$

$$\tilde{C}_L^{\text{DIS-R}}(z) = \tilde{C}^{\text{DIS-R}}(z) - 1 - z, \quad (9b)$$

$$\begin{aligned} \tilde{C}_T^{\text{DIS-R}}(z) &= \tilde{C}^{\text{DIS-R}}(z) + \frac{1}{\epsilon}(1-z) \\ &- (1-z) \ln \left(\frac{1-z}{z} \right) - \frac{3}{2} - 2z, \end{aligned} \quad (9c)$$

where the $\tilde{C}^{\text{DIS-R}}(z)$ are defined to be the combined quantities

$$\tilde{C}^{\text{DIS}}(z) = tP(z) + C^{\text{DIS}}(z), \quad (10)$$

with $P(z)$ and $C(z)$ being replaced respectively by $\Delta P(z)$ and $C_L(z)$ et cetera, where necessary. In the first equation, for $\tilde{C}(z)$, an additional contribution $3z$ due to F_L has been included to give the correction corresponding to the use of F_2 to define $q(x)$ [13]. Moreover, in (9c) the remaining ϵ is due to the difference in splitting functions and disappears in the final expression for the full coefficient.

To extract the desired coefficients $C^{\text{DIS-R}}(z)$, the virtual corrections must now, of course, also be added. First however, note that the results for the unpolarised and helicity-dependent cases agree with previous calculations, [13] and [14] respectively. Note also that the results for the various cases are (not surprisingly) similar: while the coefficient for h_1 is a little different (owing to the finite residues of the UV divergences, which lead to different splitting functions), the IR double poles in ϵ are identical

(and in any case cancel with the virtual contributions) and the single poles themselves are, of course, to be absorbed into the scale-dependent parton densities. In particular, there is no trace of the $\frac{6z \ln^2 z}{(1-z)}$ term found in the DY coefficient.

2.3.2 Virtual-gluon contributions

The virtual contributions can be partially gleaned from the literature; however, the scalar-vertex correction remains to be evaluated and this requires a some care. The virtual and real contributions arise separately gauge invariant and a natural choice (as in [13,14] and other cited work) is the Landau gauge, where it is only the vertex correction that need be calculated. Schematically,

$$\Gamma_V^\mu(q^2) = \gamma^\mu \left[1 + \frac{\alpha_s}{2\pi} \delta_V \right], \quad (11a)$$

and

$$\Gamma_S(q^2) = \mathbb{1} \left[1 + \frac{\alpha_s}{2\pi} \delta_S \right], \quad (11b)$$

where the $\mathbb{1}$ represents the “bare” scalar vertex. Then in MMS the following results are obtained:

$$\delta_V = C_F \left[\frac{\mu^2}{-q^2} \right]^\epsilon \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 - \frac{2}{3}\pi^2 \right], \quad (12a)$$

$$\delta_S = C_F \left[\frac{\mu^2}{-q^2} \right]^\epsilon \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \left[-\frac{2}{\epsilon^2} - 2 - \frac{2}{3}\pi^2 \right]. \quad (12b)$$

Noting that these corrections multiply $\delta(1-z)$, one immediately sees that the double poles $1/\epsilon^2$, of IR origin, cancel against the real diagrams, just as they should. However, note also that the single-pole structure is manifestly different; it is this point that will now be discussed.

There is a substantial difference between a vector and a scalar vertex: the former is related to a conserved current, the latter not. Thus, the vector current is not renormalised while the scalar does receive radiative corrections. In other words, one must also take into account the renormalisation of the coupling constant (i.e., the quark mass in the true Higgs case) associated with the vertices in consideration.³ Indeed, the simplest way to evaluate the contribution is to calculate the renormalisation of the quark mass, including the constant pieces. In the MMS scheme the standard calculation gives

$$\delta_m = -C_F \frac{\alpha_s(Q^2)}{2\pi} \left[\frac{\mu^2}{-q^2} \right]^\epsilon \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \frac{3}{\epsilon} \frac{1}{1-2\epsilon}. \quad (13)$$

Including this as a contribution the virtual corrections, one finally obtains

$$\delta_S^{\text{full}} = \delta_V. \quad (14)$$

³ This observation was made by Blümlein [33] in regard of similar calculations aimed at evaluating the evolution kernel.

Thus, combining real and virtual contributions, the complete coefficients for DIS are

$$\begin{aligned}
 C^{\text{DIS}}(z) &= C_F \left\{ (1+z^2) \left[\frac{\ln(1-z)}{1-z} \right]_+ - \frac{3}{2} \frac{1}{(1-z)_+} \right. \\
 &+ 3 + 2z - (1+z^2) \frac{\ln z}{(1-z)} \\
 &\left. - \left[\frac{9}{2} + \frac{\pi^2}{3} \right] \delta(1-z) \right\}, \quad (15a)
 \end{aligned}$$

$$C_L^{\text{DIS}}(z) = C^{\text{DIS}}(z) - 1 - z, \quad (15b)$$

$$\begin{aligned}
 C_T^{\text{DIS}}(z) &= C^{\text{DIS}}(z) \\
 &- \frac{3}{2} - 2z - (1-z) \ln \left(\frac{1-z}{z} \right). \quad (15c)
 \end{aligned}$$

2.4 The K -factor results

The DY and DIS coefficients can now be combined to provide a theoretical K factor. Note that in the required difference the DIS coefficient appears with a factor two; this merely reflects the two quarks (or rather quark–antiquark) in the initial state for DY. The final results are⁴

$$\begin{aligned}
 C^{\text{DY}}(z) - 2C^{\text{DIS}}(z) &= C_F \left\{ 2(1+z^2) \left[\frac{\ln(1-z)}{1-z} \right]_+ + \frac{3}{(1-z)_+} \right. \\
 &\left. - 6 - 4z + \left[\frac{4}{3}\pi^2 + 1 \right] \delta(1-z) \right\}, \quad (16a)
 \end{aligned}$$

$$\begin{aligned}
 C_L^{\text{DY}}(z) - 2C_L^{\text{DIS}}(z) &= C^{\text{DY}}(z) - 2C^{\text{DIS}}(z) + C_F 2(1+z), \quad (16b)
 \end{aligned}$$

$$\begin{aligned}
 C_T^{\text{DY}}(z) - 2C_T^{\text{DIS}}(z) &= C^{\text{DY}}(z) - 2C^{\text{DIS}}(z) \\
 &+ C_F \left\{ 7 - \frac{6z \ln^2 z}{(1-z)} - 2(1-z) \ln(1-z) \right\}, \quad (16c)
 \end{aligned}$$

where the origins of the large differences in the last line may thus be traced in part to the different phase-space restrictions in the transversity case and in part to the residues due to the different splitting functions. It is perhaps worth reminding the reader that the bulk of the large K factor in the unpolarised and helicity cases (the π^2 terms) comes from the necessary continuation of Q^2 from space-like (in DIS) to time-like (in DY). However, the transversity correction contains other non-negligible pieces.

For a first visual comparison, Fig. 3 shows the Mellin moments, defined by

$$f^{(n)} \equiv \int_0^1 dx x^n f(x), \quad (17)$$

of the above differences in the Wilson coefficients between DY and DIS for the three leading-twist densities. While it

⁴ A brief review of the results presented here may be found in a recent contributed talk [34].

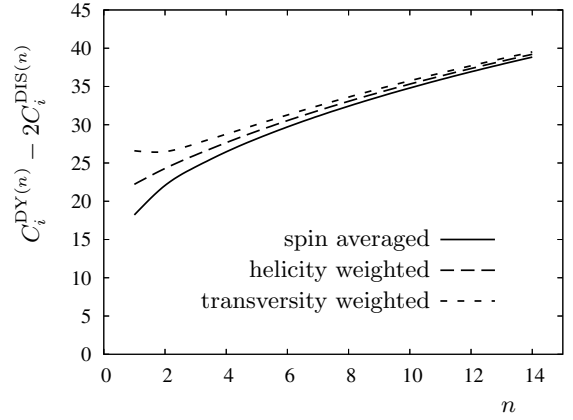


Fig. 3. The DY–DIS difference in the Mellin moments of the Wilson coefficients (i.e., the K factor) for the three leading-twist densities

is clear that the difference is generally rather more than twice as large for transversity, the growing difference as $n \rightarrow 0$ (i.e., equivalently in z space for $z \rightarrow 0$) is particularly striking.

To estimate the effect these corrections might have on a real DY asymmetry, in Fig. 4 the helicity and transversity asymmetries for purely NS contributions (both in the numerator and denominator) are shown as functions of τ . For the transversity distribution we have taken a starting point of $\Delta_T q(x, Q_0^2) = \Delta q(x, Q_0^2)$. The evolution of the distributions has then been performed to LO here as the coefficient differences are scheme independent and the effect of higher orders on the K factor is negligible. The size of the shift due to the K factor is nearly three times as large in the case of transversity with respect to helicity and typically reaches values of the order of 15%. It should also be noted, however, that there is an automatic limitation of the K -factor difference (with respect to large values of α_s , e.g., for low energies) owing to the presence of the π^2 terms; when the K -factor difference between numerator and denominator becomes large in absolute terms so too do the overall K factors themselves, which to some extent cancels or dilutes the effect.

2.5 Cross-checks

A couple of simple cross-checks may be made on the influence of the scalar vertex itself. First of all, there is now the possibility of a purely Higgs, unpolarised, DIS process (i.e., in which both vertices are scalar). The contribution of the real diagrams is

$$\tilde{C}^{\text{DIS-R,S}}(z) = \tilde{C}^{\text{DIS-R}}(z) - 2 - 3z, \quad (18)$$

which, combined with the virtual corrections already discussed, gives

$$C^{\text{DIS,S}}(z) = C^{\text{DIS}}(z) - 2 - 3z. \quad (19)$$

We may use the new coefficient in (19) in place of the usual unpolarised correction to *define* the parton distributions.

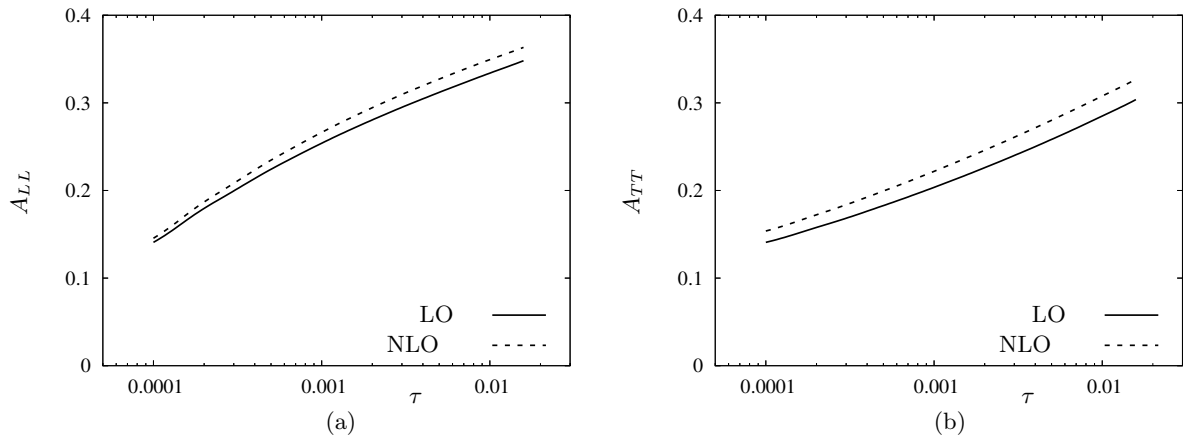


Fig. 4. The doubly polarised $p\bar{p}$ helicity and transversity asymmetries A_{LL} and A_{TT} for purely valence-driven DY at LO and NLO as functions of $\tau = Q^2/s$ (for $s = 1600 \text{ GeV}^2$)

Secondly, there is also similarly a possible purely scalar DY-like process; the NLO correction to the unpolarised cross-section in this case is found to be

$$C^{\text{DY},S}(z) = C^{\text{DY}}(z) + C_F 2(1-z). \quad (20)$$

Moreover, Hikasa’s theorem is avoided here owing to the presence of the scalar vertices and a transverse-spin asymmetry is present even after integrating over the lepton-pair azimuthal angle. The NLO correction to the scalar transversity asymmetry is

$$C_T^{\text{DY},S}(z) = C^{\text{DY}}(z) + C_F(1-z)[4 - 4\ln(1-z) + 2\ln z]. \quad (21)$$

In all cases the scalar vertex does not introduce large correction differences with respect to the vector vertex. Indeed, in the last case of the purely scalar DY process (both spin-averaged and with transverse polarisation), the only differences are residues of the difference in the LO splitting function, as indicated by the form and the overall factor $(1-z)$.

3 Conclusions

In order to appraise the real nature of the DY K factor in the case of transversity asymmetries, we have examined gedanken processes involving scalar vertices. This allows for a natural DIS definition for the partonic densities $\Delta_T q(x)$. Such a definition allows an immediate connection to be made with model estimates based on knowledge of parton densities derived essentially from precisely DIS. Typical examples might be models in which at some low Q^2 scale transversity- and helicity-weighted densities are naturally equal or others in which the Soffer bound [35] is found to be saturated, again at some low scale. In all such cases the spin-averaged and helicity-weighted densities used to set the starting point are obviously and naturally taken directly from DIS.

The results presented here provide a measure of the reliability of model predictions, without, of course, representing a rigorous estimate, in as much as the reference

processes are partially fictitious and in any case are not precisely those normally adopted. However, we have seen that in general the corrections are not excessively large although they may be significantly larger than in the helicity case. Moreover, comparison of the Mellin moments indicates that in kinematical configurations in which low $z = \tau/x_1x_2$ dominates there could be very important corrections. On the other hand, many of the differences between NLO coefficients vanish numerically for $z \rightarrow 1$ and so safe kinematical configurations certainly exist.

In closing then, although apparently fairly well under control, the question of NLO perturbative corrections in the case of transversely polarised DY processes clearly deserves more study, in particular, where the kinematics might be such experimentally as to favour the dangerous low- z region.

Acknowledgements. All loop calculations were performed with the aid of FORM version 3.1 [36]. The author would like to thank Johannes Blümlein for very useful conversations in the past concerning the calculation of transversity anomalous dimensions via photon–scalar interference diagrams.

Appendix A Plus-regularised distributions

The so-called “plus-regularised” distributions are defined via integrals with a smooth test function $f(y)$:

$$\begin{aligned} & \int_x^1 dy f(y) \left[\frac{g(y)}{1-y} \right]_+ \\ & \equiv \int_x^1 dy \left[\frac{f(y) - f(1)}{1-y} \right] g(y) - f(1) \int_0^x dy \frac{g(y)}{1-y}, \end{aligned} \quad (\text{A.1})$$

where $g(y)$ is well-behaved as $y \rightarrow 1$.

Appendix B

Modified minimal subtraction scheme: implementation

The MS scheme is defined, in conjunction with DR, as the removal of all simple poles in $1/\epsilon$ (double and higher poles due to IR divergences are cancelled automatically between real and virtual contributions). However, common residual finite contributions are always left due to the appearance of the factor $(4\pi)^\epsilon \Gamma(\epsilon)$. Expanding to $O(\epsilon)$, one obtains

$$\begin{aligned} (4\pi)^\epsilon \Gamma(\epsilon) &\simeq \Gamma(1 + \epsilon) [1 + \epsilon \ln(4\pi)] \frac{1}{\epsilon} \\ &\simeq \frac{1}{\epsilon} + \ln(4\pi) - \gamma_E. \end{aligned} \quad (\text{B.1})$$

The MMS scheme then augments MS by subtracting the two ϵ -independent terms above.

Thus, MMS may be implemented to $O(\alpha_s)$ by defining the Feynman virtual momentum-integral measure $[d^n k]$ to include a factor $1/\Gamma(1 + \epsilon)$ and by removing a factor $(4\pi)^\epsilon$. In other words, the plain MS definition may be substituted with the following:

$$[d^n k] \equiv \frac{1}{(4\pi)^\epsilon \Gamma(1 + \epsilon)} \int \frac{d^n k}{(2\pi)^n}. \quad (\text{B.2})$$

Consequently, the definition of the phase-space integral for a final two-body state (as in $\gamma q \rightarrow qg$) must be modified analogously to

$$\text{PS}_2 \equiv \frac{1}{8\pi} s^{-\epsilon} \int_0^1 dy [y(1-y)]^{-\epsilon}, \quad (\text{B.3})$$

where $y = \frac{1}{2}(1 + \cos \theta)$ and θ is the CM scattering angle (in this case between the incoming q and outgoing g).

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