

# The IUP Journal of Financial Economics

Vol. IX No. 4

Decei

# Contents

Focus



Strategic Urban Development Under Uncertainty

Flavia Cortelezzi, Pierpaolo Giannoccolo and Giovanni Villani

Foreign Aid Inflows and the Real Exchange Rate: Are There Dutch Disease Effects in Ghana?

Aliyu Rafindadi Sanusi

Impact of Political Regime and Economic Openness on Income Inequality: A Tale of Low-Income and OECD Countries Archana Pillai

A Note on the Social Costs of Monopoly and Regulation Jamal Ibrahim Haidar



Vol. IX No. 4

December 2011

# EDITOR

E N Murthy

## MANAGING EDITOR

GRK Murty

# CONSULTING EDITOR

Vishwanathan Iyer

### EDITORIAL TEAM

S V Srirama Rao (Associate Editor)

R Venkatesan lyengar

Aparajitha Sengupta

T Sri Jyothi

G Swarna

N Durga

### DIRECTOR (PRODUCTION)

H Sitaram

### Send your feedback to

The Editor, IUP Publications, # 53, Nagarjuna Hills, Panjagutta Hyderabad 500082, Andhra Pradesh, India

Tel: +91(40)23430448 E-mail: info@jupindia.in Website: www.iupindia.in

# Subscriptions

### For subscriptions and related enquiries write to:

The Executive, Subscriptions

IUP Publications

# 52, Nagarjuna Hills, Panjagutta

Hyderabad 500082, Andhra Pradesh, India

Tel: +91(40)23430448 E-mail: sery@apindia.in

### ISSN 0972-9151

The IUP Journal of Financial Economics is published four times a year in March, June, September and December.

# O December 2011. All Rights Reserved.

- No part of this publication may be reproduced or copied in any form by any means without prior written permission.
- IUP holds the copyright to all articles contributed to its publications. In case of reprinted articles, IUP holds the copyright for the selection, sequence, introduction material, summaries and other value additions.
- The views expressed in this publication are purely personal judgments of the authors and do not reflect the views of IUP. The views expressed by external authors represent their personal views and not necessarily the views of the organizations they represent.
- All efforts are made to ensure that the published information is correct. IUP is not responsible for any errors caused due to oversight or otherwise.

Subscription Rates				
	India ₹		US \$	
	By Post	By Courier	By Post	By Courier
1 year	625	7.05	3.2	40
3 years	1650	1890	90	114
5 years	2000	2400	125	165

Payment to be made by crossed Demand Draft drawn in favor of "TUCF A/c IUP", Hyderabad. Visa and MasterCard holders can subscribe online or fax their subscription order indicating their credit card number, name, amount, expiry date and signature.

Printed at M/s. ICIT Software Center Pvt. Ltd., Plot Nos. 165 & 166 P, Phase-V, IDA, Jeedimetla, Hyderabad

# Focus

The literature on real estate applications using real options is based on a pridentical firms and homogeneous products. The first paper, "Strategic Development Under Uncertainty", evaluates the equilibrium strategies developers in the real estate market, when demands are asymmetric. The paper m investment decision of a leader firm as a dynamic stochastic game and solves for cooperative case as well as the perfectly cooperative case. In case of asymmetric derimodel postulates conditions under which the follower firms would prefer to wait. The also highlight the conditions under which inefficiencies could occur in a non-concequilibrium. Possible extensions of the theoretical framework such as modeling using player dynamic game and equilibrium strategies in an incomplete information frame also indicated.

Moving on, the second paper, "Foreign Aid Inflows and the Real Exchange Rate: In Dutch Disease Effects in Ghana?", evaluates the possible Dutch disease effects of inflows on the Real Exchange Rate (RER) of Ghana. Specifically, the paper exaction of the Real Exchange Rate (RER) of Ghana. Specifically, the paper exaction of the Real Exchange Rate (RER) of Ghana. Specifically, the paper exaction to the Real Exchange Rate (RER) of Ghana. Specifically, the paper exaction to the Real Exchange Rate (RER) of Ghana. Specifically, the paper exaction to the Real Exchange Rate: In the long rate and fundamental variables capital inflows, technological change, exports, government spending, openness to the foreign assets and terms of trade. The short-run relationships are evaluated using correction model. The results indicate a peculiar situation in Ghana where increase it aid leads to some appreciation in RER in the long run, while there is no impact in run. It is the government spending which has maximum impact on RER. Most im the depreciation of the nominal exchange rate appeared to have a depreciating impact in the immediate short run, thereby suggesting that the pass-through of the nominal rate changes to inflation is substantial for Ghana.

The third paper, "Impact of Political Regime and Economic Openness on Inequality: A Tale of Low-Income and OECD Countries", analyzes the impact of regime and economic openness on income inequality in a cross-country framework. 'coefficient, a measure of income inequality, is modeled using democracy inde intensity, GDP per capita and FDI, apart from lagged Gini coefficient. Comparing th for both OECD and low-income countries, the paper notes that the more the defeatures of a regime, the more it is attuned to equity and this could lead to less inequality over a longer period of time. Trade and GDP per capita do seem to inequality, albeit at higher levels. Interestingly, the Kuznets hypothesis does not seem for low-income countries. Finally, the role of FDI was found to be weak across speci. This analysis was found to be consistent across different specifications and for vary periods. These results were further corroborated using kernel fit (a nonparametric a wherein the underlying functional form is not specified and is left to the data to fit

for itself. This result, therefore, validates the necessity of using political environment variable in studies of globalization or welfare economics.

The last paper, "A Note on the Social Costs of Monopoly and Regulation", while emphasizing that the deadweight loss discussed extensively in the literature should be considered as a lower limit in estimating the real social costs of monopoly, highlights that the existing measures are crude indicators at best. To conclude, it highlights three important assumptions along with indications for a much-needed correction in estimation of economic value of social costs of monopoly.

Vishwanathan lyer Consulting Editor

# Strategic Urban Development Under Uncertainty

Flavia Cortelezzi\*, Pierpaolo Giannoccolo\*\* and Giovanni Villani\*\*\*

The aim of this paper is to analyze the equilibrium strategies of two develop in the real estate market, when demands are asymmetric. In particular, paper considers three key features of the real estate market. First, the cos redeveloping a building is, at least partially, irreversible. Second, the levels for different buildings vary stochastically over time. Third, dem functions for space are interrelated and may produce positive or nega externalities. Using the method of option pricing theory, the paper addrethis issue at three levels. First, it models the investment decision of a firm a preassigned leader as a dynamic stochastic game. Then, it solves for the recooperative case, and for the perfectly cooperative case, in which redevelops of an area is coordinated between firms. Finally, it analyzes the efficient inefficiency of the equilibria of the game. It is found that if one firm has significantly large comparative advantage, the preemptive threat from the rewill be negligible. In this case, short burst and overbuilding phenomena, predicted by Grenadier (1996), will occur only as a limiting case.

# Introduction

In recent years, it has become possible to observe new tendencies of tourish Although in the past many major tourist destinations devoted a significant part of th supply to the so-called mass tourism<sup>4</sup>, which fundamentally involves a price computategy, nowadays many mature tourist destinations are considering specializing tourism. In fact, from mass tourism may emerge congestion problems that generat externalities to a specific area (i.e., the quality of the environment and the setting), only lower the welfare level of residents, but also affect negatively the tourism

Assistant Professor, Department of Law, Economics and Culture, Università dell'Insubria, Vi 61-22100, Como, Italy; and is the corresponding author. E-mail: flavia.cortelezzi@uninsubria

<sup>\*\*</sup> Assistant Professor, Department of Economics, Università di Bologna, Piazza Scarvilli, 1, 401. Italy. E-mail: pierpaolo.giannoccolo@unibo.it

<sup>\*\*\*</sup> Researcher, Department of Business Economics, Legal, Commodity and Geographic Sciences, t Foggia, Via Romolo Caggese 1, 71100 Foggia, Italy. E-mail: g.villani@junifg.it

This is the case of the so-called second-generation European mass tourism destinations, defined high tourist density, which have been developing in the Mediterranean region since the midwidely accepted that these destinations are in the stagnation or post-stagnation phase of Butler't 2000) tourism product life cycle model, i.e., close to their maximum carrying capacity level, and a exhausting their potential for tourists' growth. Butler (1997) also links the concept of carrying capidea of sustainable development.

production function. In response to these problems arises the need to implement a new strategic planning which allows destinations to be rejuvenated or reoriented (Butler, 1980; and Agarwal, 1994). Proposals suggest aiming for higher quality services, for example, by eliminating low-category hotels/residences and replacing them with higher category ones in a ratio lower than one unit. In other words, urban planning becomes one of the most critical determinants of success of a tourist destination. Transposing these two aspects into the real estate market framework, we can summarize that the selection of the nature of a tenant's business<sup>2</sup> and the general tastes of tenants are the two most important features that characterize an area, and consequently, the (re)development decisions are influenced by the quality type and the general characteristics of the area.

This paper is a contribution to the area of strategic urban planning and investment under uncertainty. Real estate investments are often difficult to reverse, and the timing and consequences of investment in real estate market are key strategic decisions both for developers and residents. How should a competitive real estate developer decide between waiting in developing an area and investing at once? How should he value the different options? What are the impacts of the mix of building types on the developer's investment decisions and, consequently, on the urban planning?

The present paper attempts to provide an answer to the above questions. We begin our analysis observing that in a real estate market, comparative advantages of firms in real estate investment are differentiated by their pricing rather than cost containment strategies. Therefore, by using different inverse demand functions for firms in the model, comparative advantages of firms and their effects on optimal timing in equilibrium can be explicitly examined. We develop a continuous time stochastic oligopoly model to analyze the sequence of events which originates a new urban area and use it to investigate the interaction of the various forces which may delay or anticipate markets creation. We find the conditions that may lead the ones or the others to prevail. Moreover, we analyze the perfectly cooperative case, in which (re)development activity is coordinated between firms, and use it to analyze the efficiency/inefficiency of the implemented strategies.

In our model, we consider the investment decision of a landlord intending to (re)develop his fraction of building in an area. The investment will create a new urban area (e.g., a new tourist destination), which can be interpreted as a new market, and the developer will be the market pioneer. We assume, for simplicity, that the investment is in a single new project and the investment expenditure is known and fixed, but once made it is irreversible. The demand for space of each tenant type considered has two essential features. First, the rent levels vary stochastically over time, reflecting market conditions. Second, the demands for space are interrelated. The use of space by one tenant may give rise to either positive or negative externalities for other tenants. A positive interaction between tenant types would increase the landlord's demand for a 'diversified mix' of tenants (e.g., a shopping center or

different tourist packages). Conversely, a negative externality effect would occur when of space by one type of tenant impinges upon the efficient use of space by another of heavy industrial use with residential or commercial use would be such an examp the developer must take into account both the rent levels and the interaction e choosing its optimal investment policy.

We focus on two different economic settings. We first consider the case of a landle is able to promote the development of a new area. The developer, as the pioneer resulting market, may choose the timing of the investment without bothering abortential entrants (or, in other words, may act as a pre-designated leader) and furthermore an extreme first-mover advantage which forces any subsequent entrant to the role of follower in the dynamic game. The rationale behind such modeling assist that there are many economic instances in which long-run first-mover advantage naturally. The second economic setting we investigate is a situation where two develop both potentially invest and thus begin the development of a new area. Neither firm be absolutely sure to be the first to enter the market, and strategic considerations represumably play a significant role. This second modeling strategy is intended to decompetitive situation characterized by inferior market pioneering advantages.

The general methodology adopted in the paper is that of stochastic stopping tim (Dutta and Rustichini, 1991), whilst our basic assumptions can be contrasted with Sweeney (1974), Smets (1991), Williams (1991), Dixit and Pindyck (1994), Grenadie Oflaherty (1996), Glaeser and Gyourko (2001), Irwin and Bockstael (2004), and Pc (2008).

Concerning the area of urban planning, the role of the quality type of the neight been first studied by the filtering housing market models, which deal with the deterorcess that modifies the house quality through time. These models define a hor commodity, having distinct physical characteristics and subject to urban amenitic characteristics distinguish each property from the other. According to these modecision to maintain a house in its original quality depends on the comparison of construction costs and maintenance costs. The deterioration process can be stoppe house is properly maintained. If it is not, it filters down through the price-quality (see Sweeney, 1974; and Oflaherty, 1996). Moreover, the properties' price4 is also it by the quality of the area around the building, its location and the different urban that can be accessed such as beaches, parks, shopping malls, railways, underground and employment centers. As properties are normally fixed in space, their physical characteristics.

<sup>&</sup>lt;sup>2</sup> By the term tenant type, we refer not only to the obvious distinction between the usual classes of tenants (i.e., office, retail, industrial, residential), but also to the more subtle features which distinguish one space user from another.

Alternatively, it can be argued that under uncertainty larger firms have relative advantage in making cocredibly and are inclined to move first, while smaller firms prefer to move second (see, e.g., Ha 1998). With firms of different size, it seems therefore reasonable to model the outcome of oligi-Stackelberg equilibrium.

<sup>4</sup> Rosen (1974) describes the housing market as a hedonic market. Because each house price correstied package of characteristics, the singular price of each characteristic is not observable. In that a hedonic regression links the property price to its attributes. The estimated attributes coefficients and their hedonic prices.

and location attributes are inseparable. Glaeser and Gyourko (2001) translate the spatial pattern for the decision to rebuild as a spatial pattern for the decision not to maintain a house, once it has reached the minimum quality accepted by the market. That would add a locations component to the filtering models, bringing together the logic of both urban economy and housing market models (see also Polo et al., 2008). Using parcel data on residential land conversion, Irwin and Bockstael (2004) investigate how land use externalities influence the rate of development and modify policies designed to manage urban growth and preserve open space.

Furthermore, concerning the investment under uncertainty literature, Williams (1991) focuses on the distinguishing features of a real estate market and develops a model of strategic interactions between developers. These features are summarized in the following: (1) Each real asset produces goods or services that consumers demand with a finite elasticity; (2) The rate at which assets can be developed is limited by developers capacity; (3) The supply of undeveloped assets is limited; and (4) The ownership of undeveloped assets can be monopolistic, oligopolistic or competitive. The significance of these properties results in the optimal exercise policy and in the market values of real estate versus financial assets and derives an equilibrium set of exercise strategies for real estate developers, where equilibrium development is symmetric and simultaneous. He makes a new methodological point in the real option literature applied to the real estate market. In contrast to the standard literature, Williams identifies a region of optimal exercise, replacing the single point of optimal exercise in all previous models of real options. Grenadier (1996) uses a duopolistic game theoretic approach to options exercise to explain how irrational overbuilding was induced by rapid development cascades in a volatile market. Developers are characterized by two symmetric demands and they are indifferent as to who will take the role of a leader and/or a follower; fearing the preemption by competitors, proceed into a market equilibrium in which all development occurs during a market downturn. He identifies the causes of periods of irrational overbuilding in the interaction between the fear of preemption and the time to build. Compared to Williams, in Grenadier's model, equilibrium development may arise endogenously as either simultaneous or sequential. Although this literature has made a great step towards a better understanding of investment decisions, the contribution of the real option literature to the understanding of the real estate market is still limited.

The paper is organized as follows: It is devoted to the setup of the model, and the specification will serve for the subsequent analysis. The analysis is performed with reference to a duopolistic market in which the leader is preassigned, i.e., it enjoys an extreme first-mover advantage (which allows the pioneer to dominate the market) is considered. After deriving the value of pursuing both the leader and the follower strategy, firms' investment behavior is derived. The paper presents the case of competition without preemption, i.e., a more limited effect in favor of the first entrant (with pioneer and follower competing under the same conditions) is analyzed, and subsequently provides the analysis of a cooperative solution that

will be used to identify the efficiency/inefficiency of the various market structures Finally, the conclusion is offered.

# Real Estate Market Development

In this section, we present and analyze in some detail the setup of a simple con model of irreversible investment to better understand the implications of the real e described above. Let us consider two real estate developers, denoted by  $i=L,\ I$  respectively a fraction w and 1-w of buildings in a town<sup>5</sup>; thus the total number is normalized to 1. Both owners have the opportunity to redevelop their propertie superior buildings or change their final destination. In this case, they can earn greater rentals. Thus, each owner holds an option to develop. We assume L to be i.e., the one who first exercises his development option, and F to be the follower to develop has an exercise price equal to I, the cost of construction assumed to b both firms. Moreover, to keep matters as simple as possible, we assume that I is continued and (re)development has no operating cost<sup>6</sup>. Initially, building rents,  $\overline{R}_{I}$ , are

$$\overline{R}_L = \overline{R}w$$

$$\overline{R}_F = \overline{R} \left( 1 - w \right)$$

i.e., the medium rent  $\overline{R}$ , weighted by the market share of each developer. The exert development option will result in a repercussion on the option exerciser as well a building owner. The leader, L, pays an initial construction cost and loses current the existing buildings. New buildings yield potentially higher rental rate, accordingly inverse demand function, as follows:

$$R_z = \theta_i - \alpha w$$

whose corresponding profit function is therefore:

$$\pi_L = [\theta_t - \alpha w] w$$

It represents the leader's profits for new/redeveloped buildings, where  $\alpha \geq 0$  reproved own quantity effects. We assume that the demand parameter  $\theta$  follows the geometric motion.

$$d\theta_t = \theta_t \mu dt + \theta_t \sigma d\varpi$$

where  $\mu \le r$  is the instantaneous expected growth rate of the market<sup>a</sup>,  $\sigma \ge 0$  is the instantaneous and  $d\varpi$  is an increment of a Wiener process. Thus,  $d\varpi$  is distributed accordingly.

In what follows, to simplify the analysis, we assume  $w = \frac{1}{2}$ .

<sup>\*</sup> Unlike Pawlina and Kort (2006), there are no cost asymmetries between the two developers to keep tractable.

This formulation is characterized by evolving uncertainty that comes from the demand sid redevelopment activity occurs.

<sup>\*</sup> The restriction ensures that there is a positive opportunity cost to holding the option to redevelor

normal distribution with zero mean and variance dt. It follows that the market demand curve is subject to aggregate shocks so that the developer knows current demand conditions but cannot predict future changes. This option exercise also affects the fortunes of the follower. The competitor's construction of an improved building can either improve or lessen the demand for the existing building and his profit becomes:

$$\pi_F = \overline{R}_F + \eta w (1 - w)$$

where  $\eta$  represents the one-way effect of the leader's investment on the follower's profit. It can be the effect of shops, where a different mix of shops in a borough permits convenient shopping for customers and also increases the rents of houses.

Let us now consider the impact of the follower's exercise of the development option on both owners. The follower pays the cost of construction, loses current rent, and begins receiving rents on the new (or improved) buildings. The leader is also affected by the follower's investment because he can now profit from the interrelations of tenant types. After the follower has invested, the profit functions will be:

$$\pi_L = [\theta - \alpha(w)](w) + \varepsilon_1 w (1 - w)$$

$$\pi_F = \left[\theta - \alpha (1 - w)\right] (1 - w) + \varepsilon_2(w) (1 - w)$$

where  $\varepsilon_i$  and  $\varepsilon_j$  indicate the interrelations among developers. Negative  $\varepsilon_i$  (i=1,2) denotes, for example, the case of tenant types which interact unfavorably, e.g., a mix of heavy industrial use with residential or commercial use. In this contest, the two developers are assumed to serve the same geographical market. They can, for example, develop residential projects on two land parcel within a local market. This will make strategic competition between the two developers more meaningful and relevant<sup>in</sup>.

# Value Functions

The value of the investment options for the leader, L, and follower, F, is derived in this section, taking into consideration their profit functions. As usual in dynamic games, the game is solved backwards in a dynamic programming fashion.

The Follower's Problem: Let us first value the payoff of being a follower, denoted by  $F(\theta)$ . It has three different components holding over different ranges of  $\theta$ . The first,  $F_{\theta}(\theta)$ , describes the value of investment before the leader has invested; the existing space yields a profit  $\overline{R}_F$ 

per unit of time and its present discounted value is  $\frac{R_F}{r}$ . Moreover, the follower holds the option to redevelop the existing space for the new one, conditional to the leader having

already invested. The option to invest should be valued accordingly. Let us then as the leader has already redeveloped his property, and the follower has now to e redevelopment strategy to maximize his options value. This is the second region, it value of the follower can be characterized as a portfolio containing the existing

yielding a profit  $\overline{R}_F + \eta\left(\frac{1}{4}\right) > 0$  per unit of time and a present discounted value of  $\frac{\Lambda}{2}$  plus an option to exchange the existing properties with the new one. Finally, in region,  $F_A(\theta)$ , paying an irreversible adoption cost I, the follower can redevelop hi and obtain an instantaneous profit  $\left[\left(\theta - \alpha\left(\frac{1}{2}\right)\right)\left(\frac{1}{2}\right) + \varepsilon_2\left(\frac{1}{4}\right)\right]$ , with a present discounted

$$\left[\frac{\theta\left(\frac{1}{2}\right)}{r-\mu} + \left(\frac{1}{4}\right)\frac{\varepsilon_2 - \alpha}{r}\right]$$
. Let us first derive the second and the third region. In order to

follower's optimal investment rule, notice that at each point in time the follower invest, and take the termination payoff, or can wait for an infinitesimal time dt and the decision. The payoff of the second strategy consists of the profit flow during tin dt plus the expected discounted capital gain. Denoting by  $F_{r,t}(\theta)$  the option value the Bellman equation of the problem is:

$$F_{F,1}(\theta) = Max \left\{ \frac{\overline{R}_F + \eta \left( \frac{1}{4} \right)}{r}, \frac{1}{1 + rdt} E \left[ F_{F,1}(\theta + d\theta \mid \theta) \right] \right\}$$

where E denotes the expectation operator.

Prior to investment the firm holds the opportunity to invest. It receives a p  $\overline{R}_F + \eta\left(\frac{1}{4}\right)$ , but it may experience a capital gain or loss on the value of this opt Hence, in the continuation region, i.e., the RHS of Equation (2), the Bellman equation value of the investment opportunity,  $F_{r,i}(\theta)$ , is given by:

$$rF_{F,1} dt = E \left( dF_{F,1} \right)$$

Expanding  $dF_{\nu,i}$  using Ito's lemma, we can write:

$$dF_{F,1} = F_{F,1}^{*}(\theta) d\theta + \frac{1}{2} F_{F,1}^{*}(\theta) (d\theta)^{2}$$

Substituting from Equation (1), we can write:

$$E\left(dF_{F,i}\right) = \mu \theta F_{F,i}^*(\theta) dt + \frac{1}{2} \sigma^2 \theta F_{F,i}^*(\theta) dt$$

After some simple substitutions, the Bellman equation entails the following sea differential equation:

Positive η denotes tenant types which interact favorably. It can also be negative. In this case, it denotes tenant type which interacts unfavorably. In both cases, η must be large enough to ensure positive π.

Without imposing these local market and spatial considerations, for example, in a case where developers are involved in developing an industry and an office in two different cities, the strategic interactions between the two developers could be remote, if decisions of any one of the developers will ever affect the entry or exit of his competitor.

$$\frac{1}{2} \sigma^2 \theta F_{F,1}^*(\theta) + \mu \theta F_{F,1}^*(\theta) - r F_{F,1} = 0 \qquad ...(6)$$

From Equation (1), it can be seen that if  $\theta$  ever goes to zero it stays there forever. Therefore, the option to invest has no value when  $\theta = 0$ ;  $F_{y,j}(\theta)$  must satisfy the following boundary condition:

$$F_{\nu_1}(0) = 0$$
 ...(7)

The general solution for the differential Equation (6) is:

$$F_{F,1}(\theta) = B_1 \theta^{\beta} + B_2 \theta^{\beta}$$
 ...(8)

where  $\beta > 1$  and  $\lambda < 0$  are respectively the positive and the negative root of the fundamental characteristic equation  $Q(z) = \frac{1}{2}\sigma^2 z(z-1) + \mu z - r$ , and  $B_1$  are unknown constants to be determined.

Imposing the boundary condition (7) the value of the option to invest is:

$$F_{F,j}(\theta) = B_j \theta^{jj}$$
 ...(9)

and the option value of waiting is  $F_1(\theta) = \frac{\overline{R}_F + \eta\left(\frac{1}{4}\right)}{r} + B_1\theta^\beta$ . The first part of  $F_1(\theta)$  is the expected value of the firm if the firm would never invest and the second part is the option value to invest derived above. The value in the first region is derived in the same way. The value of the option to invest is  $F_{F\beta} = B_\alpha\theta^\beta$ , and the expected value of the firm if it would never

invest is  $\frac{\overline{R}_F}{r}$ . Summing up these two components gives  $F_0(\theta) = \frac{\overline{R}_F}{r} + B_0 \theta^{\beta}$ , that is the option value of waiting in the first region.

We next consider the value of the firm in the stopping region, in which the value of  $\theta$  is such that it is optimal to invest at once. This is the third region,  $F_1(\theta)$ . Since investment is irreversible, the value of the agent in the stopping region is given by the expected value alone with no option value terms. The value of the follower adopting the new technology is given by the following expression:

$$F_2(\theta) = E \left[ \int_t^{+\infty} \left( \left[ \theta - \alpha \left( \frac{1}{2} \right) \right] \left( \frac{1}{2} \right) + \varepsilon_2 \left( \frac{1}{4} \right) \right) e^{-r \cdot (\tau - t)} d\tau - t \left( \frac{1}{2} \right) \right] \dots (10)$$

that is,

$$F_2(\theta) = \frac{\theta\left(\frac{1}{2}\right)}{r - \mu} + \left(\frac{1}{4}\right) \frac{\varepsilon_2 - \alpha}{r} - I\left(\frac{1}{2}\right) \qquad ...(11)$$

The boundary between the continuation region and the stopping region is gi trigger point  $\theta_r$  of the stochastic process such that continued delay is optimal for  $\theta$  immediate investment is optimal for  $\theta \geq \theta_r$ . The optimal stopping time is then defi first time that the stochastic process  $\theta$  hits the interval  $[\theta_r, \infty)$  from below. Puttir the three regions gives the follower's value function,  $F(\theta)$ :

$$F(\theta) = \begin{cases} \frac{\overline{R}_F}{r} + B_0 \theta^{\beta} & \theta < \theta_L \\ \frac{\overline{R}_F + \eta \left(\frac{1}{4}\right)}{r} + B_1 \theta^{\beta} & \theta \in [\theta_L, \theta_F) \\ \frac{\theta \left(\frac{1}{2}\right)}{r - \mu} + \left(\frac{1}{4}\right) \frac{\varepsilon_2 - \alpha}{r} - I\left(\frac{1}{2}\right) & \theta \ge \theta_F \end{cases}$$

Following Dixit and Pindyck (1994), the value matching and smooth pasting are used to determine the critical value describing the boundary between the contin stopping regions, along with the unknown coefficient  $B_i$ . This condition require components of the follower's value function to meet smoothly at  $\theta_p$  with a derivatives, which together with the value matching condition implies:

$$\begin{cases} \overline{R}_F + \eta\left(\frac{1}{4}\right) \\ r \\ \end{cases} + B_1 \theta_F^\beta = \frac{\theta_F \left(\frac{1}{2}\right)}{r - \mu} + \left(\frac{1}{4}\right) \frac{\varepsilon_2 - \alpha}{r} - I\left(\frac{1}{2}\right) \\ \beta B_1 \theta_F^{\beta - 1} = \left(\frac{1}{2}\right) \frac{1}{r - \mu} \end{cases}$$

Solving the above system, we can compute the value of the unknown  $B_1$  and t trigger point  $\theta_c$ :

$$\begin{split} B_1 &= \left(\frac{1}{2}\right) \frac{1}{r - \mu} \cdot \frac{1}{\beta} \cdot \theta_F^{1 - \beta} \\ \theta_F &= \frac{\beta}{\beta - 1} \cdot \frac{(r - \mu)}{r} \cdot \left[\overline{R} + \eta\left(\frac{1}{2}\right) + \alpha\left(\frac{1}{2}\right) - \varepsilon_3\left(\frac{1}{2}\right) + rI\right] \end{split}$$

It is important to note that the optimal trigger point  $\theta_r$  is not influence complementarity of developments ( $\varepsilon_1$  and  $\varepsilon_2$ ).

**Proposition 1:** Conditional on the leader having redeveloped his propertion optimal follower strategy is to invest the first moment that  $\theta_r$  equals or exceptrigger value  $\theta_r$ , as defined in Equation (14). That is, the optimal entry time follower;  $T_p$ , can be written as:

<sup>11</sup> See Dixit and Pindyck (1994), pp. 142-143, for details.

$$T_{F}=\inf\left\{t\geq0:\theta\geq\frac{\beta}{\beta-1}\cdot\left(r-\mu\right)\cdot\left[\frac{\overline{R}+\eta\left(\frac{1}{2}\right)}{r}+\left(\frac{1}{2}\right)\frac{\left(\alpha-\varepsilon_{2}\right)}{r}+I\right]\right\} ...(15)$$

The value of the unknown constant  $B_a$  is found by considering the impact of the leader's investment on the payoff to the follower. When  $\theta_t$  is first reached, the leader invests and the follower's payoff is altered either positively or negatively. Since the value functions are forward-looking,  $F_a(\theta)$  anticipates the effect of the leader's action and must therefore meet  $F_t(\theta)$  at  $\theta_t$ . Hence, a value matching condition holds at this point; however there is no optimality on the part of the follower, and so no corresponding smooth pasting condition. This implies that:

$$\frac{\overline{R}_F}{r} + B_0 \theta_L^{\beta} = \frac{\overline{R}_F + \eta \left(\frac{1}{4}\right)}{r} + B_1 \theta_L^{\beta}$$

$$B_0 = \left(\frac{1}{4}\right) \frac{\eta}{r} \theta_L^{-\beta} + \left(\frac{1}{2}\right) \frac{1}{r - \mu} \cdot \frac{1}{\beta} \cdot \theta_F^{1-\beta} \qquad ...(16)$$

Note that  $\theta_r$  is independent of the point at which the leader invests: given that the firm invests second, the precise location of the leader's trigger point is irrelevant. However, it is inversely related to the magnitude of the spillover caused by the leader investment. The effect of uncertainty is standard from the real option theory. Since  $\frac{\partial \beta}{\partial \sigma} > 0$  a greater uncertainty induces an higher trigger value. By simple substitution, the value of being the follower is thus given by the following expression:

$$F(\theta) = \begin{cases} \frac{\overline{R}_F}{r} + \left(\frac{1}{4}\right) \frac{\eta}{r} \left(\frac{\theta}{\theta_L}\right)^{\beta} + \left(\frac{1}{2}\right) \frac{1}{r - \mu} \cdot \frac{1}{\beta} \cdot \theta_F \left(\frac{\theta}{\theta_F}\right)^{\beta} & \theta < \theta_L \\ \frac{\overline{R} + \eta \left(\frac{1}{2}\right) \overline{\left(\frac{1}{2}\right)}}{r} + \left(\frac{1}{2}\right) \frac{1}{r - \mu} \cdot \frac{1}{\beta} \cdot \theta_F \left(\frac{\theta}{\theta_F}\right)^{\beta} & \theta \in [\theta_L, \theta_F) \\ \left(\frac{1}{2}\right) \frac{\theta}{r - \mu} \cdot \left(\frac{1}{4}\right) \frac{\alpha}{r} + \left(\frac{1}{4}\right) \frac{\varepsilon_2}{r} - \left(\frac{1}{2}\right) I & \theta \ge \theta_F \end{cases} ...(17)$$

The Leader's Problem: The value of the leader, denoted by  $L(\theta)$ , can be expressed as

$$L(\theta) = \begin{cases} \frac{\overline{R}\left(\frac{1}{2}\right)}{r} + B_{L0}\,\theta^{\beta} & \theta < \theta_{L} \\ \left(\frac{1}{2}\right)\frac{\theta}{r - \mu} - \left(\frac{1}{4}\right)\frac{\alpha}{r} + B_{L1}\,\theta^{\beta} - \left(\frac{1}{2}\right)I & \theta \in [\theta_{L}, \theta_{F}) \\ \left(\frac{1}{2}\right)\frac{\theta}{r - \mu} + \left(\frac{1}{4}\right)\frac{\varepsilon_{1}}{r} - \left(\frac{1}{4}\right)\frac{\alpha}{r} - \left(\frac{1}{2}\right)I & \theta \geq \theta_{F} \end{cases}$$

where  $B_{is}$  and  $B_{i,i}$  are the coefficients of the option value to invest. Sta Equation (18) one can observe that when  $\theta_i$  is first reached, the follower invest leader's expected flow payoff is altered. Since value functions are forward-lool anticipates the effect of the follower's action and must therefore meet  $L_i(\theta)$  at  $\theta_i$  value matching condition holds at this point; however there is no optimality on the leader, and so no corresponding smooth pasting condition. This implies:

$$\left(\frac{1}{2}\right)\frac{\theta}{r-\mu} - \left(\frac{1}{4}\right)\frac{\alpha}{r} + B_{L1}\theta^{\beta} - \left(\frac{1}{2}\right)I = \left(\frac{1}{2}\right)\frac{\theta}{r-\mu} + \left(\frac{1}{4}\right)\frac{\varepsilon_1}{r} - \left(\frac{1}{4}\right)\frac{\alpha}{r} - \left(\frac{1}{2}\right)I$$

that gives

$$B_{L1} = \left(\frac{1}{4}\right) \frac{\varepsilon_1}{r} \theta_F^{-\beta}$$

The usual value matching and smooth pasting conditions at the optimallydetermine the other unknown variables:

$$\begin{split} \left\{ \left(\frac{1}{2}\right) \frac{\overline{R}}{r} + B_{L0} \, \theta^{\beta} = \left(\frac{1}{2}\right) \frac{\theta}{r - \mu} - \left(\frac{1}{4}\right) \frac{\alpha}{r} + B_{L1} \, \theta^{\beta} - \left(\frac{1}{2}\right) I \\ \beta \, B_{L0} \, \theta^{\beta - 1} = \left(\frac{1}{2}\right) \frac{1}{r - \mu} + \beta \, B_{L1} \, \theta^{\beta - 1} \end{split} \right.$$

Solving the system, we can compute the value of the unknown  $B_{\ell \phi}$  and the optime point  $\theta_{\ell}$ :

$$\theta_t = \frac{\beta}{\beta - 1} \frac{r - \mu}{r} \left[ \overline{R} + \left( \frac{1}{2} \right) \alpha + rI \right]$$

Similar to the optimal trigger point  $\theta_i$ ,  $\theta_i$  is not influenced by the complement developments ( $\varepsilon_i$  and  $\varepsilon_i$ ).

$$B_{L0} = \left(\frac{1}{2}\right) \frac{1}{\beta} \left[\frac{1}{r - \mu}\right] \theta_L^{1-\beta} + \left(\frac{1}{4}\right) \frac{\varepsilon_1}{r} \theta_r^{-\beta}$$

The following appropriate account to the

**Proposition 2:** Conditional on roles exogenously assigned, the optimal leader strategy is to redevelop his properties the first moment that  $\theta_i$  equals or exceeds the trigger value  $\theta_i$ , as defined in Equation (20). That is, the optimal entry time of the leader,  $T_i$ , can be written as:

$$T_L = \inf t \ge 0$$
:  $\theta_L = \frac{\beta}{\beta - 1} \cdot \frac{r - \mu}{r} \left[ \overline{R} + \left( \frac{1}{2} \right) \alpha + rI \right]$  ...(22)

Putting together the three regions derived above by simple substitution we are able to write the leader's value function:

$$L(\theta) = \begin{cases} \left(\frac{1}{2}\right) \frac{\overline{R}}{r} + \left(\frac{1}{2}\right) \frac{1}{\beta} \left[\frac{1}{r - \mu}\right] \theta_L \left(\frac{\theta}{\theta_L}\right)^{\beta} + \frac{\varepsilon_1 w \left(1 - w\right)}{r} \left(\frac{\theta}{\theta_F}\right)^{\beta} & \theta < \theta_L \\ \left(\frac{1}{2}\right) \frac{\theta}{r - \mu} - \left(\frac{1}{4}\right) \frac{\alpha}{r} + \left(\frac{1}{4}\right) \frac{\varepsilon_1}{r} \left(\frac{\theta}{\theta_F}\right)^{\beta} - \left(\frac{1}{2}\right) t & \theta \in [\theta_L, \theta_F) \\ \left(\frac{1}{2}\right) \frac{\theta}{r - \mu} + \left(\frac{1}{4}\right) \frac{\varepsilon_1}{r} - \left(\frac{1}{4}\right) \frac{\alpha}{r} - \left(\frac{1}{2}\right) t & \theta \ge \theta_F \end{cases} ...(23)$$

In short, Propositions 1 and 2 define respectively the optimal entry time of the leader and the follower. It is worth noticing that the optimal entry timing of the follower is positively affected by the interaction effect  $\eta$  and negatively affected by  $\varepsilon_2$ . Furthermore, in order to have  $\theta_{\varepsilon} \ge \theta_{\varepsilon}$  it must be that:

$$\eta > \varepsilon_{\gamma}$$

i.e., the interaction effect of  $\eta$  has to be higher than that of  $\varepsilon_2$ . In this case, a unique sequential equilibrium exists. Otherwise, an investment cascade might occur.

# Equilibrium with Preemption

Let us now assume that the role of the leader and that of the follower are determined endogenously. As before, let us assume that one firm (the preemptor) invests strictly before the other. The follower's value function and his trigger point are the same as for the model without preemption (see Equation 14). The leader's value function is as described in the previous section. However, without the ability to precommit to a defined investment strategy at the beginning of the game, the leader's investment trigger cannot be derived as the solution to a single-agent optimization problem. This means the leader can no longer choose its investment point optimally, as if the roles were preassigned. Instead, the first firm to invest does so at the point at which it prefers to lead rather than follow. Hence, the investment point, denoted in what follows  $\theta_{pe}$  is defined by the indifference between leading and following as follows:

$$V_L(\theta_P) - I = F_F(\theta_P)$$

Proposition 3: If  $\varepsilon_1 - \varepsilon_2 > 0$  then a unique endogenous equilibrium outcome  $\theta_p$  with the following properties:

$$V_L(\theta) - I \le F_p(\theta)$$
 for  $\theta \le \theta_p$ 

$$V_{L}(\theta) - I = F_{p}(\theta)$$
 for  $\theta = \theta_{p}$ 

$$V_{\perp}(\theta) - I \ge F_{I}(\theta)$$
 for  $\theta_{P} < \theta < \theta_{I}$ 

$$V_{L}(\theta) - I = F_{\rho}(\theta)$$
 for  $\theta = \theta_{\rho}$ 

**Proof:** Let us define the function  $\Delta(\theta) = L_1(\theta) - F_1(\theta)$ , describing the gain of where  $L_1(\theta)$  is conditional on the 'preemptor' having invested, and  $F_1(\theta)$  is the of the follower. By using Equations (18) and (9), we get

$$\Delta(\theta) = \left(\frac{1}{2}\right) \frac{\theta}{r - \mu} - \left(\frac{1}{4}\right) \frac{\alpha}{r} - \left(\frac{1}{2}\right) I + \left(\frac{1}{4}\right) \frac{\varepsilon_1}{r} \left(\frac{\theta}{\theta_F}\right)^{\beta}$$

$$- \frac{\left[\overline{R} + \eta\left(\frac{1}{2}\right)\right] \left(\frac{1}{2}\right)}{r} - \left(\frac{1}{2}\right) \frac{1}{r - \mu} \cdot \frac{1}{\beta} \cdot \theta_F \left(\frac{\theta}{\theta_F}\right)^{\beta}$$

First, we establish the existence of a root for  $\Delta(\theta)$  in the interval  $(0, \theta_0)$ . E

$$\begin{split} \theta &= 0 \quad \text{yields} \quad \Delta \big(0\big) = -\bigg(\frac{1}{4}\bigg)\frac{\alpha}{r} - \bigg(\frac{1}{2}\bigg)I - \frac{\bigg[\overline{R} + \eta\left(\frac{1}{2}\right)\bigg]\bigg(\frac{1}{2}\bigg)}{r} < 0 \;. \;\; \text{Similarly, evaluating} \\ \text{yields} \quad \Delta \big(\theta_r\big) &= \bigg(\frac{1}{4}\bigg)\frac{\left(\varepsilon_1 - \varepsilon_2\right)}{r}, \;\; \text{i.e.,} \end{split}$$

$$\Delta(\theta_F) > 0$$
 if  $(\varepsilon_1 - \varepsilon_2) > 0$ 

Therefore,  $\Delta(\theta)$  must have at least one root in the interval  $(0, \theta_r)$ . Finalgebraic manipulation yields  $\Delta'(0) = \left(\frac{1}{2}\right) \frac{1}{r-\mu} > 0$  and  $\lim_{\theta \to \theta_r} \Delta(\theta)' = +\left[\left(\frac{1}{4}\right)\beta \, \varepsilon_1 \frac{1}{r} - \left(\frac{1}{2}\right) \frac{1}{r-\mu} \, \theta_r\right] (\theta_r)^{-1} \geq 0$  if  $\varepsilon_1 \geq 0$ . To prove uniqueness, one mere demonstrate strict concavity (convexity) over the interval. Differentiating  $\Delta(\theta)$  to  $\Delta''(\theta) = (\beta - 1) \left[\beta \, \varepsilon_1 \, \frac{1}{4r} - \frac{1}{2 \, (r-\mu)} \, \theta_r\right] \left(\frac{\theta}{\theta_r}\right)^{\theta-2} \geq 0$  if  $\theta_r \geq A^{12}$ . Thus, the root is

In order to simplify notation, let us define  $A = \left(\frac{1}{2}\right) \beta \frac{r}{r - \mu} \kappa_1$ .

**Proposition 4:** If  $\varepsilon_t - \varepsilon_2 \le 0$  and  $\varepsilon_t \ge 0$ , then  $V_{\varepsilon}(\theta) - 1 \le F_{\varepsilon}(\theta)$  for all  $\theta \le \theta_F$ . An endogenous equilibrium outcome does not exist in the interval  $(0, \theta_{\varepsilon})$ .

Proposition 5: If  $\varepsilon_i - \varepsilon_2 \le 0$  and  $\varepsilon_i \le 0$ , then,

- a. If  $\Delta(\theta^*) < 0$  then an endogenous equilibrium does not exist;
- b. If  $\Delta(\theta^*_p) = 0$  then a unique endogenous equilibrium  $\theta^*_p \in (0, \theta_p)$  exists with the following properties:

$$V_{i}(\theta) - I \le F_{i}(\theta)$$
 for  $\theta \le \theta$ .

$$V_i(\theta) - I = F_i(\theta)$$
 for  $\theta = \theta$ .

$$V_s(\theta) - I \ge F_s(\theta)$$
 for  $\theta < \theta < \theta$ 

$$V_{\underline{t}}(\theta) - I = F_{\underline{t}}(\theta)$$
 for  $\theta = \theta_{\underline{t}}$ 

c. If  $\Delta(\theta^*_{p}) \ge 0$  then two equilibria  $\theta^*_{p_1}$  and  $\theta^*_{p_2} \in (0, \theta_p)$  exist with the following properties:

$$V_i(\theta) - I \le F_i(\theta)$$
 for  $\theta \le \theta_{ij}^*$ 

$$V_i(\theta) - I = F_i(\theta)$$
 for  $\theta = \theta^*$ 

$$V_i(\theta) - I \ge F_i(\theta)$$
 for  $\theta_{ij}^* < \theta < \theta_{ij}^*$ 

$$V_l(\theta) - I = F_s(\theta)$$
 for  $\theta = \theta_s^*$ 

$$V_{l}(\theta) - l \le F_{p}(\theta)$$
 for  $\theta_{p,2}^{*} \le \theta < \theta_{p}$ 

$$V_{i}(\theta) - I = F_{i}(\theta)$$
 for  $\theta = \theta_{i}$ 

**Proof:** Let us define the function  $\Delta(\theta) = L_1(\theta) - F_2(\theta)$ , describing the gain of preemption, where  $L_1(\theta)$  is conditional on the 'preemptor' having invested, and  $F_1(\theta)$  is the option value of the follower. By using Equations (18) and (9), we get

$$\Delta(\theta) = \left(\frac{1}{2}\right) \frac{\theta}{r - \mu} - \left(\frac{1}{4}\right) \frac{\alpha}{r} - \left(\frac{1}{2}\right) I + \left(\frac{1}{4}\right) \frac{\varepsilon_1}{r} \left(\frac{\theta}{\theta_F}\right)^{\beta}$$

$$-\frac{\left[\overline{R} + \eta\left(\frac{1}{2}\right)\right]\left(\frac{1}{2}\right)}{r} - \left(\frac{1}{2}\right)\frac{1}{r - \mu} \cdot \frac{1}{\beta} \cdot \theta_{F}\left(\frac{\theta}{\theta_{F}}\right)^{\beta} \qquad ...(26)$$

First, we establish the existence of a root for  $\Delta(\theta)$  in the interval  $(0, \theta_r)$ . Ev

$$\theta = 0 \text{ yields } \Delta(0) = -\left(\frac{1}{4}\right)\frac{\alpha}{r} - \left(\frac{1}{2}\right)I - \frac{\left[\overline{R} + \eta\left(\frac{1}{2}\right)\right]\left(\frac{1}{2}\right)}{r} < 0 \text{ . Similarly, evaluating :}$$

yields 
$$\Delta(\theta_r) = \left(\frac{1}{4}\right) \frac{(\varepsilon_1 - \varepsilon_2)}{r}$$
, i.e.,

$$\Delta(\theta_F) < 0$$
 if  $(\varepsilon_1 - \varepsilon_2) < 0$ 

Finally, some algebraic manipulation yields  $\Delta'(0) = \left(\frac{1}{2}\right) \frac{1}{r-\mu} > 0$ 

$$\lim_{\theta \to \theta_r} \Delta(\theta)' = \left(\frac{1}{2}\right) \frac{1}{r-\mu} \quad + \left[\left(\frac{1}{4}\right)\beta \varepsilon_1 \frac{1}{r} - \left(\frac{1}{2}\right) \frac{1}{r-\mu} \theta_F\right] (\theta_F)^{-1} \quad \gtrless \quad 0 \quad \text{if} \quad \varepsilon_i \quad \gtrless \quad 0.$$

uniqueness, one merely needs to demonstrate strict concavity (convexity) over the

Differentiating 
$$\Delta(\theta)$$
 twice yields:  $\Delta''(\theta) = (\beta - 1) \left[ \beta \, \varepsilon_1 \, \frac{1}{4r} - \frac{1}{2 \, (r - \mu)} \, \theta_r \, \right] \left( \frac{\theta}{\theta_r} \, \right)^{\beta - 2}$  easy to prove that:

- If  $\varepsilon_1 > 0$  then  $\lim_{\theta \to \theta_2} \Delta(\theta)' > 0$  and if  $\theta_p < A$  then  $\Delta''(\theta) > 0$ ; this implies the equilibrium does not exist;
- If  $\varepsilon_i \ge 0$  then  $\lim_{\theta \to \theta_p} \Delta(\theta)' > 0$  and if  $\theta_p \ge A$  then  $\Delta^*(\theta) < 0$ ; this implies the equilibrium does not exist;
- If ε<sub>i</sub> < 0 then lim<sub>θ→θ<sub>i</sub></sub> Δ(θ) < 0 and Δ\*(θ) < 0 ∀ θ ∈ (0, θ<sub>i</sub>); def θ\*<sub>p</sub> = arg max Δ(θ) we get that if Δ(θ\*<sub>p</sub>) < 0 then an endogenous equilibrium not exist; if Δ(θ\*) = 0 then a unique endogenous equilibrium θ\*<sub>p</sub> ∈ (0 exists; and finally if Δ(θ\*) > 0 then there exist two endogenous equilibria θ\*<sub>p,i</sub>
   ∈ (0, θ<sub>p</sub>), Q.E.D.

Propositions 3, 4 and 5 show the effect of an interrelated demand for space. The real option effect is that the first investor's trigger point  $\theta_c$  is greater than the strategi  $\theta_r$ , due to uncertainty and irreversibility (see Propositions 3 and 5b). We can find the result in Grenadier (1996). He concludes that in this case it would be optimal for the totake a preemptive move to reap higher payoff; as a consequence a short be overbuilding phenomena might occur. However, introducing asymmetric demands impure Grenadier (1996) will occur only as a limiting case. In fact, Propositions 4, 5a and 5c cases not considered in Grenadiers model that reduces his findings to a particular Specifically, in Propositions 4 and 5a conditions for nonexistence of equilibria are. This means that there is a strong incentive to follow rather than to lead. Moreover,  $P_{DD}$  5c exploits the range of parameter values in 15.1.

equilibria or a nonexistence of equilibria. It is worth noticing that if  $\theta \ge \theta_p$  it is always better to invest.

# Cooperative Solution

This section analyzes the cooperative solution, in which the agents' investment trigger points are chosen to maximize the sum of their two value functions. The objective is to provide a benchmark to identify inefficiencies.

Let us examine the case in which investment is sequential. Two trigger points,  $\theta_{tL}$  and  $\theta_{2L}$ , are chosen to maximize the sum of the leader's and follower's value functions, denoted by  $C_{t,tL}(\theta)$ . Using the same steps as before, it is given by:

$$C_{L+F}(\theta) = \begin{cases} \frac{\overline{R}}{r} + B_0 \theta^{\beta} + B_1 \theta^{\beta} & \theta < \theta_{2L} \\ \frac{\overline{R}}{r} + q \left(\frac{1}{2}\right) - \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right) - \frac{\theta}{r - \mu} - \left(\frac{1}{4}\right) \frac{\alpha}{r} + B_2 \theta^{\beta} - \left(\frac{1}{2}\right) I + B_3 \theta^{\beta} & \theta \in [\theta_{2L}, \theta_{2F}) \\ \frac{\theta}{r - \mu} - \left(\frac{1}{4}\right) \frac{\alpha}{r} - \left(\frac{1}{4}\right) \frac{\alpha}{r} - I + \left(\frac{1}{4}\right) \frac{(\varepsilon_1 + \varepsilon_2)}{r} & \theta \ge \theta_{2F} \end{cases} ...(28)$$

where  $B_i$ , i=0,1,2,3 are constants. The cooperative trigger points are determined by the value matching and smooth pasting conditions at both points. Solving the system, we get the leader trigger and the follower trigger points,  $\theta_{iL}$  and  $\theta_{SF}$  respectively, given by:

$$\begin{split} & \frac{\left[\left(\overline{R} + \eta\left(\frac{1}{2}\right)\right)}{r} \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right) \frac{\theta}{r - \mu} - \left(\frac{1}{4}\right) \frac{\alpha}{r} + B_2 \theta^{\beta} - \left(\frac{1}{2}\right) I + B_3 \theta^{\beta} \\ &= \frac{\theta}{r - \mu} - \left(\frac{1}{4}\right) \frac{\alpha}{r} - \left(\frac{1}{4}\right) \frac{\alpha}{r} - I + \left(\frac{1}{4}\right) \frac{(\varepsilon_1 + \varepsilon_2)}{r} \\ &= \left(\frac{1}{2}\right) \frac{1}{r - \mu} + \beta \left(B_2 + B_3\right) \theta^{\beta + 1} = \frac{1}{r - \mu} \end{split}$$

$$\theta_{2F} = \left(\frac{\beta}{\beta - 1}\right) \frac{r - \mu}{r} \left[\overline{R} + \left(\frac{1}{2}\right) \eta + \left(\frac{1}{2}\right) \alpha + rI - \left(\frac{1}{2}\right) (\varepsilon_1 + \varepsilon_2)\right] \qquad ...(29)$$

$$\begin{split} & \frac{\overline{R}}{r} + B_0 \theta^{\beta} + B_1 \theta^{\beta} = \underbrace{\left(\overline{R} + \eta \left(\frac{1}{2}\right)\right)}_{r} \left(\frac{1}{2}\right) \\ & + \left(\frac{1}{2}\right) \frac{\theta}{r - \mu} - \left(\frac{1}{4}\right) \frac{\alpha}{r} + B_2 \theta^{\beta} - \left(\frac{1}{2}\right) I + B_3 \theta^{\beta} \\ & \beta \left(B_0 + B_1\right) \theta^{\beta - 1} = \beta \left(B_2 + B_3\right) \theta^{\beta - 1} + \left(\frac{1}{2}\right) \frac{1}{r - \mu} \end{split}$$

$$\theta_{tt} = \left(\frac{\beta}{\beta - 1}\right) \frac{(r - \mu)}{\tau} \left[\overline{R} - \eta \left(\frac{1}{2}\right) + \alpha \left(\frac{1}{2}\right) + rI\right]$$

Equations (29) and (30) identify the trigger values of the leader and the follower, are affected by the interaction effect and externalities. Let us now analyze inefficience may arise in the non-cooperative equilibria by comparing Equations (29) and (Equations (20) and (14). Two main results arise:

- By maximizing the sum of the two value functions, the cooperative leader ta into account the role of the interaction effect (η). In particular, in presence of positive (negative) interaction effect, the non-cooperative leader starts to deve too late (early) with respect to the optimum (θ<sub>ii</sub> < (>) θ<sub>i</sub>)<sup>13</sup>.
  - 2. By maximizing the sum of the two value functions, the cooperative follower only is positively affected by the interaction effect η and negatively affected ε<sub>2</sub>, but is also negatively affected by the effect that the leader suffers when the develop. In particular, in the presence of negative (positive) externalities, the necooperative follower starts to develop too early (late) with respect to the optim (θ<sub>10</sub> > (<) θ<sub>2</sub>).

Summarizing, the presence of interaction effects and externalities influences the tin (re)development driving the individual choices to no optimal equilibria. In particul absence of cooperation implies that the agents anticipate or delay the optimal entry ti these cases, an important role for the central authority arises and it is possible to i optimal decisions that move the market into a Pareto-optimum level. The first best d for the central authority is to coordinate the individual decisions and reach the optima time for all developers (e.g., by coordinating and programming a vast redevelopm exindustrial area, or by changing a mass tourist area into a specialized one). In this is possible to take into account the role of the interaction effects and internalize the p externalities that arise when developers operate in the same area. When a direct control developers' decisions is not possible, it is possible to identify some second best polici help the agents to come close to the optimal level. In general, these policies that i implemented may have direct and indirect effects. Direct policy effects influent developers' decisions by directly influencing either the costs or returns to developmen by constraining land use choices and improving the level of public service infrastructures in the area). These policies, referring to the model, act directly on the of the developers.

Furthermore, there are the indirect policy effects which influence the externalitie neighbors and so, indirectly, the development value of the land. These policies, refer the model, act on  $\eta$ ,  $\varepsilon$  and, indirectly, on the payoffs of the developers.

Note that the optimum is relative to the cooperative benchmark. In this analysis we do not consciousmer welfare aspects of the problem.

# Conclusion

Although the real options literature has made a great step towards a better understanding of investment decisions, the contribution of the real option literature to the understanding of the real estate market is still limited. Williams (1991) and Grenadier (1996) were among few researchers who introduced this methodology in real estate applications. However, in these models, firms are assumed to be identical and products are homogeneous. This symmetric assumption restricts the application of such models only to selected cases.

In this paper, we relax the symmetric hypothesis and analyze the equilibrium strategies of two developers in the real estate market, when demands are asymmetric. We therefore extend the standard 'real option' analysis to a setting where there are general strategic interactions between agents. In particular, we assume demand functions for space are interrelated and may produce either positive or negative externalities. In symmetric demand models, equilibrium strategies either sequential or simultaneous, are driven largely by the action of a comparatively strong leader. This result becomes a special case when we analyze an asymmetric demand. We have shown that if the follower's comparative disadvantage is much weaker than its competitor, then the follower will prefer to wait. Moreover, when one firm has a significantly large comparative advantage, the preemptive threat from the rival will be negligible.

Furthermore, we identified the regions of parameter values in which an overbuilding activity with cascading investment or a sluggish activity might occur and showed that overbuilding phenomena as predicted by Grenadier (1996) will occur only as a limiting case and waiting strategies are optimal when the comparative strengths between firms are small; firms will prefer to wait for its competitor to take the first move when the market is volatile.

Finally, we analyze the cooperative solution in which the agents' investment trigger points are chosen to maximize the sum of their two value functions. Comparing the trigger values of both firms in case of cooperation, we can analyze the inefficiencies that arise in the non-cooperative equilibrium. We find that, without cooperation, the leader does not take into account the presence of the interaction effect ( $\eta$ ) and anticipates (delays) the optimal entry time when  $\eta > 0$  ( $\eta < 0$ ) and the follower does not take into account the presence of complementarity to the leader's value function when both develop ( $\varepsilon_1$ ) and anticipates (delays) the decision to (re)develop when  $\varepsilon_1 < 0$  ( $\varepsilon_1 > 0$ ). In this case the presence of a central authority that coordinates the development decisions of the agents can avoid an irrational urban planning either internalizing the externalities or considering the role of the interaction effects. Even if a direct control of the developers decisions is not possible, it is possible to identify some second best policies that help the agents to come close to the optimal level by influencing the payoffs directly or, by implementing indirect policy effects which influence  $\eta$  and  $\varepsilon$ , indirectly.

Several extensions in improving the theoretical framework can be explored in future research. First, we can analyze a microfoundation of the model and analyze its empirical implementation. Second the duopoly game theoretic framework can be extended to include multi-player dynamic game. Finally, asymmetric equilibrium strategies can be analyzed in an incomplete information framework taking into consideration the asymmetries of firms. •

Acknowledgment: The authors are thankful to Giuseppe Colangelo, Giovanni Marseguerra, and Nicola for their helpful comments. This paper has been realized within the research program Models in Economics and Finance: Interactions, Complexity and Forecast', Financial support from Ministerodell'Università e della Ricera Scientifica e Tecnologica-COFIN-MIUR 2004-2006 is acknowledged.

# Bibliography

- Agarwal S (1994), "The Resort Cycle Revisited: Implications for Resorts", in C and A Lockwood (Eds.), Progress in Tourism, Recreation and Hospitality Ma Vol. 5, pp. 194-208, Cassell, London.
- 2. Bertola G (1989), "Irreversible Investment", Mimeo, Princeton University.
- Brealey R A and Myers S C (1992), Principle of Corporate Finance, 4th Edition, Hill, New York.
- Brennan M J and Trigeorgis L (Eds.) (2000), Project Flexibility, Age Competition: New Developments in the Theory and Application of Real Option University Press, Oxford and New York.
- Butler R W (1980), "The Concept of a Tourism Area Cycle of Evolution: Im for the Management of Resources", Canadian Geographer, Vol. 24, pp. 5-12.
- Butler R W (1997), "The Concept of Carrying Capacity for Tourism Destination or Merely Buried?", in C Cooper and S Wanhill (Eds.), Tourism Dev Environmental and Community Issues, pp. 11-21, John Wiley, Chichester.
- Butler R W (2000), "The Resort Cycle Two Decades On", B Faulkner, E Laws, G (Eds.), Tourism in the 21st Century: Reflections on Experience, pp. 284-298, London.
- Capozza D R and Yuming L (1994), "The Intensity and Timing of Investment: of Land", American Economic Review, Vol. 84, pp. 889-904.
- Capozza D R and Yuming L (2001), "Residential Investment and Interest R Empirical Test of Land Development as a Real Option", Real Estate Economics, pp. 503-519.
- Dixit A (1993), "The Art of Smooth Pasting", in Jacques Lesourne and Hugo Schein (Eds.), Fundamentals of Pure and Applied Economics, Vol. 55, Academic Publishers, Chur, Switzerland.
- Dixit A and Pindyck R S (1994), Investment Under Uncertainty, Princeton UPress, Princeton.
- Duany A, Elizabeth Plater-Zyberk and Jeff Speck (2000), Suburban Nation: The Sprawl and the Decline of the American Dream, North Point Press, New York
- Duffie D (1992), Dynamic Asset Pricing Model, Princeton University Press, P. New Jersey.

- Dutta P K and Rustichini A (1991), "A Theory of Stopping Time Game with Applications to Product Innovation and Asset Sales", Discussion Paper No. 523, Department of Economics, Columbia University.
- Glaser E and Gyourko J (2001), "Urban Decline and Durable Housing", NBER Working Paper No. 8598.
- Grenadier S R (1996), "The Strategic Exercise of Options: Development Cascades and Overbuilding in Real Estate Markets", The Journal of Finance, Vol. 5, December, pp. 1653-1679.
- Grenadier S R and Weiss A M (1997), "Investment in Technological Innovation: An Option Pricing Approach", Journal of Financial Economics, Vol. 44, pp. 397-416.
- Hay D A and Liu G S (1998), "The Investment Behaviour of Firms in an Oligopolistic Setting", The Journal of Industrial Economics, Vol. 1, pp. 79-99.
- Howell S (Ed.) (2001), Real Options: Evaluating Corporate Investment Opportunities in a Dynamic World, Financial Times/Prentice Hall, London.
- Huisman K J and Kort P (1999), "Effect of Strategic Interactions on the Option Value of Waiting", Discussion Paper, Department of Econometries and Center, Tilburg University.
- Huisman K J and Kort P (2000), "Strategic Technology Adoption Taking into Account Future Technological Improvements: A Real Option Approach", Discussion Paper, Department of Econometrics and Center, Tilburg University.
- Hull J (1989), Options, Futures, and Other Derivative Securities, Prentice-Hall, Englewood Cliffs, New Jersey.
- Irwin E G and Bockstael N E (2004), "Land Use Externalities, Open Space Preservation, and Urban Sprawl", Regional Science and Urban Economics, Vol. 34, pp. 705-725.
- Kunstler J H (1993), The Geography of Nowhere: The Rise and Decline of Americas Man-Made Landscape, Simon & Schuster, New York.
- Kunstler J H (1996), Home from Nowhere: Remaking Our Everyday-World for the Twenty-First Century, Simon & Schuster, New York.
- Mason R and Weeds H (2000), "Networks, Options and Preemptions", Working Paper, University of Southampton.
- McDonald R and Siegel D (1984), "The Value of Waiting to Invest", Quarterly Journal of Economics, Vol. 101, pp. 707-728.
- Merton R C (1976), "Option Pricing When Underlying Stock Returns are Discontinuous", Journal of Financial Economics, Vol. 3, pp. 125-144.
- Nabarro R and Key T (1992), "Current Trends in Commercial Property Investment and Development: An Overview", in Healey et al. (Eds.), Rebuilding the City, E&FN Spon, London.

- Oflaherty B (1996), Making Room: The Economics of Homelessness, Harvard U Press.
- Pawlina G and Kort P (2006), "Real Options in an Asymmetric Duopoly: Who from Your Competitive Disadvantage?", Journal of Economics & Management Vol. 15, No. 1, pp. 1-35.
- Polo Clemente, Vicente Ramos, Javier Rey-Maquieira et al. (2008), "The Potentia of a Change in the Distribution of Tourism Expenditure on Employment", Economics, Vol. 14, No. 4, pp. 709-725.
- Quigg L (1993), "Empirical Testing of Real Option-Pricing Models", Journal of Vol. 48, pp. 621-640.
- Rosen S (1974), "Hedonic Prices and Implicit Markets: Product Differentiation Competition", Journal of Political Economy, Vol. 72, pp. 34-55.
- Smets F (1991), "Exporting Versus FBI: The Effect of Uncertainty, Irreversibil Strategic Interactions", Working Paper, Yale University.
- Stokey N L and Lucas Jr. R E and Edward C Prescott (1989), Recursive Me Economic Dynamics, Harvard University Press, Cambridge, MA.
- Sweeney J (1974), "A Commodity Hierarchy Model of the Rental Housing Journal of Urban Economics, Vol. 1, pp. 288-323.
- 38. Tirole J (1988), The Theory of Industrial Organization, MIT Press, Cambridge
- 39. Trigeorgis L (1996), Real Options, MIT Press, Cambridge, MA.
- Weeds H (1998), Strategic Delay in a Real Options Model of R&D Competition, University of Warwick.
- Weeds H (1999), "Sleeping Patents and Compulsory Licensing: An Options A Warwick Economic Research, No. 577, University of Warwick.
- Williams J T (1991), "Real Estate Development as an Option", Journal of Rec Finance and Economics, Vol. 4, pp. 191-208.
- Williams J T (1997), "Redevelopment of Real Assets", Real Estate Economics, pp. 387-407.

Reference # 421-2011-

# Information to Authors

- The paper should be typed in MS Word.
- Title of the paper should be followed by name, e-mail and affiliation of author(s)
- The paper should be accompanied with a declaration to the effect that the paper is the original work of the author(s) and that the paper has not been submitted for publication
- mentioned, and the author should specify with whom the copyright rests In case of papers presented/submitted in a conference/seminar, details must be clearly
- Use a single column layout with both left and right margins justified.
- Font: Main Body-12 pt.; Style: Times, or the closest comparable font available
- Tables and Figures

should tables or figures be in a separate document or file they are referenced in the text. Avoid the use of overly small type in tables. In no case To the extent possible, tables and figures should appear in the document near/after where

- An abstract of not more than 200 words is required.
- summarizing the findings of the paper. The paper should start with an introduction and it should end with a conclusion
- References

in the following style: that ..." and the list of all the references must be placed at the end of the manuscript information. References should appear in the text as: "Gruben and Meleod (1998) reported author's obligation to provide complete references with the necessary

- Gruben W and Mcleod D (1998), "Capital Flows Savings and Growth in the 1990s", Quarterly Review of Economics and Finance, Vol. 38, No. 3, pp. 287-301.
- Lewis W A (1955), Theory of Economic Growth, Allen and Unwin Ltd., London.
- on the first page along with the title of the paper and should not be repeated unywhere To ensure anonymity, the writer's name, designation and other details should appear only Papers are processed through a blind referral system by experts in the subject areas
- All manuscripts should be in electronic form and sent to:

The Editor

The IUP Journal of Financial Economics

IUP Publications

# 53, Nagarjuna Hills, Panjagutta

Hyderabad 500082, Andhra Pradesh, India

Tel: +91(40)23430448

E-mail: info@iupindia.in; ijfineco@iupindia.in

Website: www.iupindia.in