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The literature on real estate applications using real options is based on a set of identical firms and homogeneous products. The first paper, "Strategic Development Under Uncertainty", evaluates the equilibrium strategies for developers in the real estate market, when demands are asymmetric. The paper analyzes the investment decision of a leader firm as a dynamic stochastic game and solves for cooperative case as well as the perfectly cooperative case. In case of asymmetric distribution, it is shown that the followers would prefer to wait. It also highlights the conditions under which inefficiencies could occur in a non-equilibrium model.

Moving on, the second paper, "Foreign Aid Inflows and the Real Exchange Rate: Dutch Disease Effects in Ghana?", evaluates the possible Dutch disease effects of inflows on the Real Exchange Rate (RER) of Ghana. Specifically, the paper examines the co-integrating relationship between real exchange rate and fundamental variables such as capital inflows, technological change, imports, government spending, openness to foreign assets, and terms of trade. The short-run relationships are evaluated using a correction model. The results indicate that a peculiar situation in Ghana where increase in aid leads to some appreciation in RER in the long run, while there is no impact in the short run. However, the appreciation of the nominal exchange rate has maximum impact on RER. Most importantly, the depreciation of the nominal exchange rate appeared to have a depreciating impact in the immediate short run, thereby suggesting that the pass-through of the nominal rate changes to inflation is substantial for Ghana.

The third paper, "Impact of Political Regime and Economic Openness on Income Inequality: A Tale of Low-Income and OECD Countries", analyzes the impact of regime and economic openness on income inequality in a cross-country framework. The income inequality, measured using the index of the Gini coefficient, is modeled using democracy index, GDP per capita and FDI, apart from lagged Gini coefficient. Comparing the results for both OECD and low-income countries, the paper notes that the more the features of a regime, the more it is attuned to equity and this could lead to less inequality over a longer period of time. Trade and GDP per capita do seem to have a significant impact, albeit at higher levels. Interestingly, the Kuznets hypothesis does not seem to hold for low-income countries. Finally, the role of FDI was found to be weak across specification. This analysis was found to be consistent across different specifications and for various periods. These results were further corroborated using kernel regression (a nonparametric approach) wherein the underlying functional form is not specified and is left to the data to fit.
for itself. This result, therefore, validates the necessity of using political environment variable in studies of globalization or welfare economics.

The last paper, "A Note on the Social Costs of Monopoly and Regulation", while emphasizing that the deadweight loss discussed extensively in the literature should be considered as a lower limit in estimating the real social costs of monopoly, highlights that the existing measures are crude indicators at best. To conclude, it highlights three important assumptions along with indications for a much-needed correction in estimation of economic value of social costs of monopoly.

Vishwanathan Iyer
Consulting Editor

Strategic Urban Development
Under Uncertainty

Flavia Cortezzi, Pierpaolo Giannoccolo and Giovanni Villani

The aim of this paper is to analyze the equilibrium strategies of two developers in the real estate market, when demands are asymmetric. In particular, the paper considers three key features of the real estate market. First, the cost of redeveloping a building is, at least partially, irreversible. Second, the levels for different buildings vary stochastically over time. Third, demand functions for space are interrelated and may produce positive or negative externalities. Using the method of option pricing theory, the paper addresses this issue at three levels. First, it models the investment decision of a firm as a preassigned leader in a dynamic stochastic game. Then, it solves for the cooperative case, and for the perfectly cooperative case, in which redevelopment of an area is coordinated between firms. Finally, it analyzes the efficient inefficiency of the equilibrium of the game. It is found that if one firm has a significantly large comparative advantage, the preemptive threat from the other will be negligible. In this case, short burst and overbuilding phenomena, predicted by Grenadier (1996), will occur only as a limiting case.

Introduction
In recent years, it has become possible to observe new tendencies of tourism. Although in the past many major tourist destinations devoured a significant part of their supply to the so-called mass tourism—an approach which fundamentally involves a price cut strategy—nowadays many mature tourist destinations are considering specializing tourism. In fact, from mass tourism may emerge congestion problems that generate externalities to a specific area (i.e., the quality of the environment and the setting), only lowering the welfare level of residents, but also affecting negatively the tourist

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1 This is the case of the so-called second-generation European mass tourism destinations, defined by high tourist density, which have been developing in the Mediterranean region since the mid-1980s; it is widely accepted that these destinations are in the stagnation or post-stagnation phase of Butler's (2000) tourism product life cycle model, i.e., close to their maximum carrying capacity level, and are exhausting their potential for tourism growth. Butler (1997) also links the concept of carrying capacity idea of sustainable development.
production function. In response to these problems arises the need to implement a new strategic planning which allows destinations to be rejuvenated or reoriented (Butler, 1980; and Agarwal, 1994). Proposals suggest aiming for higher quality services, for example, by eliminating low-category hotels/residences and replacing them with higher category ones in a ratio lower than one unit. In other words, urban planning becomes one of the most critical determinants of success of a tourist destination. Transposing these two aspects into the real estate market framework, we can summarize that the selection of the nature of a tenant's business and the general tastes of tenants are the two most important features that characterize an area, and consequently, the (re)development decisions are influenced by the quality type and the general characteristics of the area.

This paper is a contribution to the area of strategic urban planning and investment under uncertainty. Real estate investments are often difficult to reverse, and the timing and consequences of investment in real estate market are key strategic decisions both for developers and residents. How should a competitive real estate developer decide between waiting in developing an area and investing at once? How should value the different options? What are the impacts of the mix of building types on the developer's investment decisions and, consequently, on the urban planning?

The present paper attempts to provide an answer to the above questions. We begin our analysis observing that in a real estate market, comparative advantages of firms in real estate investment are differentiated by their pricing rather than cost containment strategies. Therefore, by using different inverse demand functions for firms in the model, comparative advantages of firms and their effects on optimal timing in equilibrium can be explicitly examined. We develop a continuous time stochastic oligopoly model to analyze the sequence of events which originates a new urban area and use it to investigate the interaction of the various forces which may delay or anticipate markets creation. We find the conditions that may lead the ones or the others to prevail. Moreover, we analyze the perfectly cooperative case, in which (re)development activity is coordinated between firms, and use it to analyze the efficiency/inefficiency of the implemented strategies.

In our model, we consider the investment decision of a landlord intending to (re)develop his fraction of building in an area. The investment will create a new urban area (e.g., a new tourist destination), which can be interpreted as a new market, and the developer will be the market pioneer. We assume, for simplicity, that the investment is in a single new project and the investment expenditure is known and fixed, but once made it is irreversible. The demand for space of each tenant type considered has two essential features. First, the rent levels vary stochastically over time, reflecting market conditions. Second, the demands for space are interrelated. The use of space by one tenant may give rise to either positive or negative externalities for other tenants. A positive interaction between tenant types would increase the landlord's demand for a 'diversified' mix of tenants (e.g., a shopping center or different tourist packages). Conversely, a negative externality effect would occur when of space by one type of tenant impinges upon the efficient use of space by another of heavy industrial use with residential or commercial use would be such an example the developer must take into account both the rent levels and the interaction and choosing its optimal investment policy.

We focus on two different economic settings. We first consider the case of a land is able to promote the development of a new area. The developer, as the pioneer resulting market, may choose the timing of the investment without bothering about potential entrants (or, in other words, may act as a pre-designated leader) and furthermore an extreme first-mover advantage which forces any subsequent entrant to the role of follower in the dynamic game. The rationale behind such modeling is that there are many economic instances in which long-run first-mover advantage naturally. The second economic setting we investigate is a situation where two developers both potentially invest and thus the development of a new area. Neither firm be absolutely sure to be the first to enter the market, and strategic considerations presumably play a significant role. This second modeling strategy is intended to dynamic situation characterized by inferior market pioneering advantages.


Concerning the area of urban planning, the role of the quality type of the neighborhood been first studied by the filtering housing market models, which deal with the decision process that modifies the house quality through time. These models define a new commodity, having distinct physical characteristics and subject to urban amenities characteristics distinguish each property from the other. According to these new decision to maintain a house in its original quality depends on the comparison of construction costs and maintenance costs. The deterioration process can be stopped if the house is properly maintained. If it is not, it filters down through the price-quality (see Sweeney, 1974; and Ofahery, 1996). Moreover, the properties' price is also by the quality of the area around the building, its location and the different urban that can be accessed such as beaches, parks, shopping malls, railways, underground and employment centers. As properties are normally fixed in space, their physical char

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1 By the term tenant type, we refer not only to the obvious distinction between the usual classes of tenants (i.e., office, retail, industrial, residential), but also to the more subtle features which distinguish one space user from another.

2 Alternatively, it can be argued that under uncertainty larger firms have relative advantage in making coherently and are inclined to move first, while smaller firms prefer to move second (see, e.g., Ha 1994). With firms of different size, it seems therefore reasonable to model the outcome of oligo-Stackelberg equilibrium.

3 Rosen (1974) describes the housing market as a hedonic market. Because each house price carries a bundle of characteristics, the singular price of each characteristic is not observable. In that a hedonic regression links the property price to its attributes. The estimated attributes coefficients are their hedonic prices.
and location attributes are inseparable. Glaeser and Gyourko (2001) translate the spatial pattern for the decision to rebuild as a spatial pattern for the decision not to maintain a house, once it has reached the minimum quality accepted by the market. That would add a locations component to the filtering models, bringing together the logic of both urban economy and housing market models (see also Polo et al., 2008). Using parcel data on residential land conversion, Irwin and Backstiel (2004) investigate how land use externalities influence the rate of development and modify policies designed to manage urban growth and preserve open space.

Furthermore, concerning the investment under uncertainty literature, Williams (1991) focuses on the distinguishing features of a real estate market and develops a model of strategic interactions between developers. These features are summarized in the following: (1) Each real asset produces goods or services that consumers demand with a finite elasticity; (2) The rate at which assets can be developed is limited by developers' capacity; (3) The supply of undeveloped assets is limited; and (4) The ownership of undeveloped assets can be monopolistic, oligopolistic, or competitive. The significance of these properties results in the optimal exercise policy and in the market values of real estate versus financial assets and derives an equilibrium set of exercise strategies for real estate developers, where equilibrium development is symmetric and simultaneous. He makes a new methodological point in the real option literature applied to the real estate market. In contrast to the standard literature, Williams identifies a region of optimal exercise, replacing the single point of optimal exercise in all previous models of real options. Grenadier (1996) uses a duopolistic game theoretic approach to options exercise to explain how irrational overbuilding was induced by rapid development cascades in a volatile market. Developers are characterized by two symmetric demands and they are indifferent as to who will take the role of a leader and/or a follower: fearing the preemption by competitors, proceed into a market equilibrium in which all development occurs during a market downturn. He identifies the causes of periods of irrational overbuilding in the interaction between the fear of preemption and the time to build. Compared to Williams, in Grenadier's model, equilibrium development may arise endogenously as either simultaneous or sequential. Although this literature has made a great step towards a better understanding of investment decisions, the contribution of the real option literature to the understanding of the real estate market is still limited.

The paper is organized as follows: It is devoted to the setup of the model, and the specification will serve for the subsequent analysis. The analysis is performed with reference to a duopolistic market in which the leader is preassigned, i.e., it enjoys an extreme first-mover advantage (which allows the pioneer to dominate the market) is considered. After deriving the value of pursuing both the leader and the follower strategy, firms' investment behavior is derived. The paper presents the case of competition without preemption, i.e., a more limited effect in favor of the first entrant (with pioneer and follower competing under the same conditions) is analyzed, and subsequently provides the analysis of a cooperative solution that will be used to identify the efficiency/inefficiency of the various market structures. Finally, the conclusion is offered.

Real Estate Market Development

In this section, we present and analyze in some detail the setup of a simple con model of irreversible investment to better understand the implications of the real estate described above. Let us consider two real estate developers, denoted by and respectively a fraction and of buildings in a town; thus the total number is normalized to 1. Both owners have the opportunity to redevelop their properties superior buildings or change their final destination. In this case, they can earn greater rents. Thus, each owner holds an option to develop. We assume to be i.e., the one who first exercises his development option, and to be the follower. To develop has an exercise price equal to , the cost of construction assumed by both firms. Moreover, to keep matters as simple as possible, we assume that is a constant and (re)development has no operating cost. Initially, building rents, are

\[
\bar{R}_L = R_{w} \\
\bar{R}_F = R(1-w)
\]

e.g., the medium rent , weighted by the market share of each developer. The exercise development option will result in a repercussion on the option exercised as well a building owner. The leader, pays an initial construction cost and loses current the existing buildings. New buildings yield potentially higher rental rate, account in the demand function, as follows:

\[
\bar{R}_L = \theta - \alpha w
\]

whose corresponding profit function is therefore:

\[
\bar{\sigma}_L = \left[\theta - \alpha w\right]w
\]

It represents the leader's profit for new/redeveloped buildings, where is the own quantity effects. We assume that the demand parameter follows the geometric motion:

\[
d\theta = \theta \mu dt + \theta \sigma dW
\]

where is the instantaneous expected growth of the market, is the variance and is an increment of a Wiener process. Thus, is distributed according to a normal distribution.

\[1\] In what follows, to simplify the analysis, we assume .

\[2\] Unlike Pavlin and Kort (2006), there are no cost asymmetries between the two developers to keep tractable.

\[3\] This formulation is characterized by evolving uncertainty that comes from the demand and redevelopment activity occurs.

\[4\] The restriction ensures that there is a positive opportunity cost to holding the option to redevelop.
normal distribution with zero mean and variance \( dt \). It follows that the market demand curve is subject to aggregate shocks so that the developer knows current demand conditions but cannot predict future changes. This option exercise also affects the fortunes of the follower. The competitor's construction of an improved building can either improve or lessen the demand for the existing building and its profit becomes:

\[
\pi_r = R_r + \eta w (1 - w)
\]

where \( \eta \) represents the one-way effect of the leader's investment on the follower's profit. It can be the effect of shops, where a different mix of shops in a borough permits convenient shopping for customers and also increases the rents of houses.

Let us now consider the impact of the follower's exercise of the development option on both owners. The follower pays the cost of construction, loses current rent, and begins receiving rents on the new (or improved) buildings. The leader is also affected by the follower's investment because he can now profit from the interrelations of tenant types. After the follower has invested, the profit functions will be:

\[
\pi_l = \left[ 1 - \alpha (u) \right] w + \varepsilon_2 (1 - w)
\]

\[
\pi_r = \left[ 1 - \alpha (1 - w) \right] (1 - w) + \varepsilon_1 (w) (1 - w)
\]

where \( \varepsilon_1 \) and \( \varepsilon_2 \) indicate the interrelations among developers. Negative \( \varepsilon_i \) (\( i = 1, 2 \)) denotes, for example, the case of tenant types which interact unfavorably, e.g., a mix of heavy industrial use with residential or commercial use. In this context, the two developers are assumed to serve the same geographical market. They can, for example, develop residential projects on two land parcels within a local market. This will make strategic competition between the two developers more meaningful and relevant.

Value Functions

The value of the investment options for the leader, \( L \), and follower, \( F \), is derived in this section, taking into consideration their profit functions. As usual in dynamic games, the game is solved backwards in a dynamic programming fashion.

The Follower's Problem: Let us first value the payoff of being a follower, denoted by \( F (\theta) \). It has three different components holding over different ranges of \( \theta \). The first, \( F_L (\theta) \), describes the value of investment before the leader has invested; the existing space yields a profit \( R_r \), per unit of time and its present discounted value is \( \frac{R_r}{r} \). Moreover, the follower holds the option to redevelop the existing space for the new one, conditional to the leader having already invested. The option to invest should be valued accordingly. Let us then as the leader has already redeveloped his property, and the follower has new to choose a redevelopment strategy to maximize his options value. This is the second region, \( \alpha \). The value of the follower can be characterized as a portfolio containing the existing unit yield a \( \frac{R_r + \eta (1)}{r} \) per unit of time and a present discounted value of \( \frac{\theta}{r} \), plus an option to exchange the existing properties with the new one. Finally, in region, \( F_L (\theta) \), paying an irreversible adoption cost \( I \), the follower can redevelop and obtain an instantaneous profit \( \left[ \theta - \alpha (\frac{1}{2}) \right] (1 - \varepsilon_1 + \varepsilon_2) \), with a present discounted value of \( \frac{\theta (1)}{r - I} + \frac{\theta \varepsilon_2 - \alpha}{r} \). Let us now derive the second and the third region. In order to the follower's optimal investment rule notice that at each point in time the follower invest, and take the termination or payoff, or can wait for an infinitesimal time \( dt \) and the decision. The payoff of the second strategy consists of the profit flow during time \( dt \) plus the expected discounted capital gain. Denoting by \( F_L (\theta, \phi) \) the option value the Bellman equation of the problem is:

\[
F_L (\theta) = \max \left( \frac{R_r + \eta (1)}{r} + \frac{1}{1 + \frac{1}{dt}} E \left[ F_L (\theta, \phi + dt, \phi) \right] \right)
\]

where \( E \) denotes the expectation operator.

Prior to investment the firm holds the opportunity to invest. It receives a \( p \frac{R_r + \eta (1)}{r} \), but it may experience a capital gain or loss on the value of this option. Hence, in the continuation region, i.e., the RHS of Equation (2), the Bellman equation value of the investment opportunity, \( F_L (\theta, \phi) \), is given by:

\[
r F_L (\theta, \phi) = E (dF_L (\theta))
\]

Expanding \( dF_L (\theta) \), using Ito's lemma, we can write:

\[
dF_L (\theta) = F_L (\theta) d\theta + \frac{1}{2} \sigma^2 F_L (\theta) (d\theta)^2
\]

Substituting from Equation (1), we can write:

\[
E (dF_L (\theta)) = \mu \theta F_L (\theta) d\theta + \frac{1}{2} \sigma^2 \theta F_L (\theta) (d\theta)^2
\]

After some simple substitutions, the Bellman equation entails the following ordinary differential equation:
The boundary between the continuation region and the stopping region is given by trigger point \( \theta_t \) of the stochastic process \( \theta \), such that continued delay is optimal for \( \theta \geq \theta_t \). The optimal stopping time is then defined as the first time that the stochastic process \( \theta \) hits the interval \([\theta_s, \infty)\) from below. Put \( \theta \) as the three regions gives the following value function, \( F(\theta) \):

\[
F(\theta) = \begin{cases} 
\frac{\tilde{R}_c}{r} + \frac{1}{4} \eta + \frac{1}{2} \theta^2 & \theta < \theta_s \\
\frac{\tilde{R}_c}{r} + \frac{1}{4} \eta + \frac{1}{2} \theta^2 + \frac{1}{4} \left( \frac{\beta}{r - \mu} \right) \left( \frac{\theta - \frac{1}{2}}{r - \mu} \right) + \frac{1}{4} \left( \frac{\beta}{r - \mu} \right) \left( \frac{\theta - \frac{1}{2}}{r - \mu} \right) & \theta \geq \theta_t 
\end{cases}
\]

Following Dixit and Pindyck (1994), the value matching and smooth pasting are used to determine the critical value describing the boundary between the continuation and stopping regions, along with the unknown coefficient \( \theta_t \). This condition requires components of the follower's value function to meet smoothly at \( \theta_t \) with its derivatives, which together with the value matching condition implies:

\[
\begin{align*}
\frac{\tilde{R}_c}{r} + \frac{1}{4} \eta + \frac{1}{2} \theta^2 &= \frac{\beta}{r - \mu} \left( \frac{1}{2} \right) + \frac{1}{4} \left( \frac{\beta}{r - \mu} \right) \left( \frac{\theta - \frac{1}{2}}{r - \mu} \right) + \frac{1}{4} \left( \frac{\beta}{r - \mu} \right) \left( \frac{\theta - \frac{1}{2}}{r - \mu} \right) \\
\beta \theta_t \theta_t^{-1} &= \frac{1}{2} \frac{1}{r - \mu}
\end{align*}
\]

Solving the above system, we can compute the value of the unknown \( \theta_t \).

\[
\theta_t = \frac{1}{2} \frac{1}{r - \mu} \beta \theta_t^{-1} \]

\[
\theta_t = \theta_t \left( \frac{1}{2} \frac{1}{r - \mu} \beta \theta_t^{-1} \right)
\]

It is important to note that the optimal trigger point \( \theta_t \) is not influenced by developments (\( \epsilon_1 \) and \( \epsilon_2 \)).

**Proposition 1** Conditional on the leader having redeveloped his property, an optimal follower strategy is to invest the first moment that \( \theta \) equals or exceeds the trigger value \( \theta_t \) as defined in Equation (14). That is, the optimal early time follower, \( T_{\theta_t} \), can be written as:

\[
T_{\theta_t} = \min \left\{ t : \theta_t = \theta_t \left( \frac{1}{2} \frac{1}{r - \mu} \beta \theta_t^{-1} \right) \right\}
\]

---

*See Dixit and Pindyck (1994), pp. 142-143, for details.*
\[ T_r = \inf \left\{ r \geq 0 : \theta \geq \frac{\beta}{\beta - 1} (r - \mu) \left[ \frac{R + \eta \left( \frac{1}{2} \left( \frac{1}{r} \right) \left( \frac{\alpha - \epsilon_r}{r} \right) \right)}{r} \right] \right\} \] (15)

The value of the unknown constant \( B_{1\theta} \) is found by considering the impact of the leader’s investment on the payoff to the follower. When \( \theta_L \) is first reached, the leader invests and the follower’s payoff is altered either positively or negatively. Since the value functions are forward-looking, \( F_L(\theta) \) anticipates the effect of the leader’s action and must therefore meet \( F_L(\theta) \) at \( \theta_L \). Hence, a value matching condition holds at this point; however, there is no optimality on the part of the follower, and so no corresponding smooth pasting condition. This implies that:

\[ \frac{R - \mu}{r} + B_{1\theta} \theta^\beta \]

\[ B_{1\theta} = \left( \frac{1}{4} \right) \frac{1}{r} \left( \frac{\alpha - \epsilon_r}{r} \right) \theta^\beta \] (16)

Note that \( \theta_L \) is independent of the point at which the leader invests; given that the firm invests second, the precise location of the leader’s trigger point is irrelevant. However, it is inversely related to the magnitude of the spillover caused by the leader investment. The effect of uncertainty is standard from the real options theory. Since \( \frac{\partial V}{\partial \epsilon} > 0 \) a greater uncertainty induces an higher trigger value. By simple substitution, the value of being the follower is thus given by the following expression:

\[ F(\theta) = \left[ \frac{R + \eta \left( \frac{1}{2} \right) \left( \frac{1}{r} \right) \left( \frac{\alpha - \epsilon_r}{r} \right)}{r} \right] \left( \frac{1}{2} \right) \theta \left[ \frac{1}{r} \right] + \left( \frac{1}{2} \right) \frac{1}{r - \mu} \beta \theta \left[ \frac{\theta}{\theta} \right] \theta \geq \theta_L \]

\[ \theta \in [\theta_L, \theta_F] \] (17)

**The Leader’s Problem:** The value of the leader, denoted by \( L(\theta) \), can be expressed as:

\[ L(\theta) = \left( \frac{1}{2} \right) \frac{\theta}{r - \mu} \left( \frac{1}{4} \right) \alpha + B_{1\theta} \theta^\beta \] 

where \( B_{1\theta} \) and \( B_{1\theta} \) are the coefficients of the option value to invest. See Equation (16) one can observe that when \( \theta_L \) is first reached, the follower invests. The leader’s expected payoff is altered. Since value functions are forward-looking, \( F_L(\theta) \) anticipates the effect of the follower’s action and must therefore meet \( L(\theta) \) at \( \theta_L \). A value matching condition holds at this point; however, there is no optimality on the part of the leader, and so no corresponding smooth pasting condition. This implies:

\[ \left( \frac{1}{2} \right) \frac{\theta}{r - \mu} \left( \frac{1}{4} \right) \alpha + B_{1\theta} \theta^\beta \]

\[ \theta \in [\theta_L, \theta_F] \]

that gives:

\[ B_{1\theta} = \left( \frac{1}{4} \right) \frac{1}{r} \theta^\beta \]

The usual value matching and smooth pasting conditions at the optimally-determine the other unknown variables:

\[ \beta B_{1\theta} \theta^\beta = \left( \frac{1}{2} \right) \frac{\theta}{r - \mu} \left( \frac{1}{4} \right) \alpha + B_{1\theta} \theta^\beta \]

\[ B_{1\theta} = \left( \frac{1}{4} \right) \frac{1}{r} \theta^\beta \]

Solving the system, we can compute the value of the unknown \( B_{1\theta} \) and the optimal point \( \theta_L \):

\[ \theta_L = \frac{\beta}{\beta - 1} \frac{r - \mu}{r} \left[ \frac{R}{\beta} \left( \frac{1}{2} \right) \alpha + r I \right] \]

Similar to the optimal trigger point \( \theta_L \), \( \theta_L \) is not influenced by the complexities developments (e.g. and \( \epsilon_r \)).

\[ B_{1\theta} = \left( \frac{1}{2} \right) \frac{1}{r} \theta^\beta \] 

The following equation is...
**Proposition 2:** Conditional on roles exogenously assigned, the optimal leader strategy is to redevelop his properties the first moment that \( \theta_l \) equals or exceeds the trigger value \( \theta_t \), as defined in Equation (20). That is, the optimal entry time of the leader, \( T_l^* \), can be written as:

\[
T_l^* = \inf \{ t \geq 0 : \theta_l = \frac{\beta\theta}{\beta - 1} \frac{r - \mu}{r} \left[ R^* \left( \frac{1}{2} \right) \alpha + \frac{1}{2} \right] \}
\]

Putting together the three regions derived above by simple substitution we are able to write the leader's value function:

\[
V_l(\theta) = \begin{cases} 
\left( \frac{1}{2} \left[ R^* \left( \frac{1}{2} \right) \alpha \left( \frac{1}{2} \right) \beta \frac{r - \mu}{r} \right] \theta_t \left( \frac{1}{2} \right) r - \mu \right) \theta^{\beta} & \theta < \theta_t \\
\left( \frac{1}{2} \left[ \frac{1}{2} \theta - \frac{1}{2} \theta r - \mu \right] \alpha, \left( \frac{1}{2} \right) \beta \frac{r - \mu}{r} \right) \theta_t \left( \frac{1}{2} \right) r - \mu \right) \theta^{\beta} & \theta \in [\theta_t, \theta_f] \\
\left( \frac{1}{2} \left[ \frac{1}{2} \theta - \frac{1}{2} \theta r - \mu \right] \alpha, \left( \frac{1}{2} \right) \beta \frac{r - \mu}{r} \right) \theta_t \left( \frac{1}{2} \right) r - \mu \right) \theta^{\beta} & \theta > \theta_f
\end{cases}
\]

In short, Propositions 1 and 2 define respectively the optimal entry time of the leader and the follower. It is worth noticing that the optimal entry timing of the follower is positively affected by the interaction effect \( \eta \) and negatively affected by \( \varepsilon_s \). Furthermore, in order to have \( \theta_f > \theta_l \), it must be that:

\[ \eta > \varepsilon_s \]

i.e., the interaction effect of \( \eta \) has to be higher than that of \( \varepsilon_s \). In this case, a unique sequential equilibrium exists. Otherwise, an investment cascade might occur.

**Equilibrium with Preemption**

Let us now assume that the role of the leader and that of the follower are determined endogenously. As before, let us assume that one firm (the precommit) invests strictly before the other. The follower's value function and his trigger point are the same as for the model without preemption (see Equation 14). The leader's value function is as described in the previous section. However, without the ability to precommit to a defined investment strategy at the beginning of the game, the leader's investment trigger cannot be derived as the solution to a single-agent optimization problem. This means that the leader can no longer choose its investment point optimally, as if the roles were preassigned. Instead, the first firm to invest does so at the point at which it prefers to lead rather than follow. Hence, the investment point, denoted in what follows \( \theta_t^* \) is defined by the indifference between leading and following. As follows:

\[
V_l(\theta_l) - l = F_l(\theta_f)
\]

**Proposition 3:** If \( \varepsilon_s - \varepsilon_f > 0 \) then a unique endogenous equilibrium outcome \( \theta_f \) with the following properties:

\[
\begin{align*}
V_l(\theta) - l & < F_l(\theta) \quad \text{for } \theta < \theta_f, \\
V_l(\theta) - l & = F_l(\theta) \quad \text{for } \theta = \theta_f, \\
V_l(\theta) - l & > F_l(\theta) \quad \text{for } \theta_f < \theta < \theta_f, \\
V_l(\theta) - l & = F_l(\theta) \quad \text{for } \theta = \theta_f.
\end{align*}
\]

**Proof:** Let us define the function \( \Delta(\theta) = L_l(\theta) - F_l(\theta) \), describing the gain of where \( L_l(\theta) \) is conditional on the 'precommit' having invested, and \( F_l(\theta) \) is that of the follower. By using Equations (18) and (19), we get

\[
\begin{align*}
\Delta(\theta) &= \left( \frac{1}{2} \right) \theta \left( \frac{1}{2} \right) \beta \frac{r - \mu}{r} \theta_t \left( \frac{1}{2} \right) r - \mu \right) \theta^{\beta} \\
&= \left( \frac{1}{2} \right) \theta \left( \frac{1}{2} \right) \beta \frac{r - \mu}{r} \theta_t \left( \frac{1}{2} \right) r - \mu \right) \theta^{\beta} \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) \beta \frac{r - \mu}{r} \theta_t \left( \frac{1}{2} \right) r - \mu \right) \theta^{\beta} \left( \frac{1}{2} \right) \theta_t \left( \frac{1}{2} \right) r - \mu \right) \theta^{\beta}
\end{align*}
\]

First, we establish the existence of a root for \( \Delta(\theta) \) in the interval \((0, \theta_f)\). E

\[
\theta = 0 \quad \text{yields } \Delta(0) = \left( \frac{1}{4} \right) \theta \left( \frac{1}{2} \right) \beta \frac{r - \mu}{r} \left( \frac{1}{2} \right) \theta_t \left( \frac{1}{2} \right) r - \mu \right) \theta^{\beta} < 0. \quad \text{Similarly, evaluating}
\]

\[
\Delta(\theta_f) > 0 \quad \text{if } (\varepsilon_s - \varepsilon_f) > 0
\]

Therefore, \( \Delta(\theta) \) must have at least one root in the interval \((0, \theta_f)\). Fix algebraic manipulation yields \( \Delta(\theta) = \left( \frac{1}{2} \right) \theta \left( \frac{1}{2} \right) \beta \frac{r - \mu}{r} \theta_t \left( \frac{1}{2} \right) r - \mu \right) \theta^{\beta} \left( \theta_f \right)^{\beta-2} \ |

| \left( \frac{1}{2} \right) \beta \varepsilon_s \left( \frac{1}{2} \right) \beta \frac{r - \mu}{r} \theta_t \left( \frac{1}{2} \right) r - \mu \right) \theta^{\beta} \left( \theta_f \right)^{\beta-2} \ |

\[
\Delta(\theta) = \left( \frac{1}{4} \right) \beta \varepsilon_s \left( \frac{1}{2} \right) \beta \frac{r - \mu}{r} \theta_t \left( \frac{1}{2} \right) r - \mu \right) \theta^{\beta} \left( \theta_f \right)^{\beta-2} \ |

\[
\Delta(\theta) = \left( \frac{1}{4} \right) \beta \varepsilon_s \left( \frac{1}{2} \right) \beta \frac{r - \mu}{r} \theta_t \left( \frac{1}{2} \right) r - \mu \right) \theta^{\beta} \left( \theta_f \right)^{\beta-2} \ |

\[
\Delta(\theta) = \left( \frac{1}{4} \right) \beta \varepsilon_s \left( \frac{1}{2} \right) \beta \frac{r - \mu}{r} \theta_t \left( \frac{1}{2} \right) r - \mu \right) \theta^{\beta} \left( \theta_f \right)^{\beta-2} \ |

\[
\Delta(\theta) = \left( \frac{1}{4} \right) \beta \varepsilon_s \left( \frac{1}{2} \right) \beta \frac{r - \mu}{r} \theta_t \left( \frac{1}{2} \right) r - \mu \right) \theta^{\beta} \left( \theta_f \right)^{\beta-2} \ |

\[
\Delta(\theta) = \left( \frac{1}{4} \right) \beta \varepsilon_s \left( \frac{1}{2} \right) \beta \frac{r - \mu}{r} \theta_t \left( \frac{1}{2} \right) r - \mu \right) \theta^{\beta} \left( \theta_f \right)^{\beta-2} \ |

\[
\Delta(\theta) = \left( \frac{1}{4} \right) \beta \varepsilon_s \left( \frac{1}{2} \right) \beta \frac{r - \mu}{r} \theta_t \left( \frac{1}{2} \right) r - \mu \right) \theta^{\beta} \left( \theta_f \right)^{\beta-2} \ |
Proposition 4: If $e_i - e_j < 0$ and $e_i < 0$, then $V_i(\theta) - I < F_j(\theta)$ for all $\theta > 0$. An endogenous equilibrium outcome does not exist in the interval $(0, \theta_r)$.

Proposition 5: If $e_i - e_j < 0$ and $e_i < 0$, then,

- If $M(\theta^*_j) < 0$ then an endogenous equilibrium does not exist;
- If $\Delta(\theta^*_j) = 0$ then a unique endogenous equilibrium $\theta^*_r \in (0, \theta_r)$ exists with the following properties:
  
  \[ V_i(\theta) - I < F_j(\theta) \quad \text{for} \quad \theta < \theta^*_r \]
  \[ V_i(\theta) - I = F_j(\theta) \quad \text{for} \quad \theta = \theta^*_r \]
  \[ V_i(\theta) - I > F_j(\theta) \quad \text{for} \quad \theta^*_r < \theta < \theta_r \]
  \[ V_i(\theta) - I = F_j(\theta) \quad \text{for} \quad \theta = \theta_r \]

- If $\Delta(\theta^*_j) > 0$ then two equilibria $\theta^*_r_1$ and $\theta^*_r_2 \in (0, \theta_r)$ exist with the following properties:
  
  \[ V_i(\theta) - I < F_j(\theta) \quad \text{for} \quad \theta < \theta^*_r_1 \]
  \[ V_i(\theta) - I = F_j(\theta) \quad \text{for} \quad \theta = \theta^*_r_1 \]
  \[ V_i(\theta) - I > F_j(\theta) \quad \text{for} \quad \theta^*_r_1 < \theta < \theta^*_r_2 \]
  \[ V_i(\theta) - I = F_j(\theta) \quad \text{for} \quad \theta = \theta^*_r_2 \]
  \[ V_i(\theta) - I < F_j(\theta) \quad \text{for} \quad \theta^*_r_2 < \theta < \theta_r \]
  \[ V_i(\theta) - I = F_j(\theta) \quad \text{for} \quad \theta = \theta_r \]

Proof: Let us define the function $M(\theta) = L_i(\theta) - F_j(\theta)$, describing the gain of preemption, where $L_i(\theta)$ is conditional on the 'preemptor' having invested, and $F_j(\theta)$ is the option value of the follower. By using Equations (18) and (9), we get

\begin{align*}
\Delta(\theta) &= (\frac{1}{2}) \frac{\theta}{r - \mu} - \frac{1}{4} \frac{\alpha}{r} - \frac{1}{2} \frac{\beta}{r} \theta \left( \frac{\theta}{\theta_r} \right)^2 \\
&\quad + \left[ \frac{R + q}{2} \right] \left( \frac{1}{2} \right) \frac{1}{r} \left( \frac{\theta}{\theta_r} \right)^2 \\
&\quad - \left( \frac{1}{2} \right) \frac{1}{r - \mu} \left( \frac{\theta}{\theta_r} \right) \beta \right] \\
&\quad \left( \frac{\theta}{\theta_r} \right)^2
\end{align*}

... (20)

First, we establish the existence of a root for $\Delta(\theta)$ in the interval $(0, \theta_r)$. For

\[ \theta = 0 \text{ yields } \Delta(0) = \left( \frac{1}{2} \right) \frac{\alpha}{r} - \left( \frac{1}{2} \right) \frac{1}{r} \left[ \frac{R + q}{2} \right] \left( \frac{1}{2} \right) \frac{1}{r} < 0 \]. Similarly, evaluating

\[ \Delta(\theta_r) = \left( \frac{1}{2} \right) \frac{e_i - e_j}{r} \left( \theta_r \right)^2 \], i.e.,

\[ \Delta(\theta_r) < 0 \text{ if } (e_i - e_j) < 0 \]

Finally, some algebraic manipulation yields

\[ \Delta(\theta) = \left( \frac{1}{2} \right) \frac{\theta}{r - \mu} - \left( \frac{1}{4} \right) \frac{1}{r} \left[ \frac{1}{2} \right] \frac{\beta}{r} \theta \left( \frac{\theta}{\theta_r} \right)^2 \]

\[ \text{lim}_{\theta \to \theta_r} \Delta(\theta) = \left( \frac{1}{2} \right) \frac{1}{r - \mu} \left[ \beta e_i \theta \left( \frac{\theta}{\theta_r} \right)^2 \right] \geq 0 \text{ if } \theta_i \geq 0 \]

uniqueness, one merely needs to demonstrate strict concavity (convexity) over the

Differentiating $\Delta(\theta)$ twice yields:

\[ \Delta'\prime(\theta) = (\theta - 1) \left[ \beta e_i \theta \left( \frac{\theta}{\theta_r} \right)^2 \right] \left( \frac{1}{\theta} \right) \theta_r \cdot 1 \]

easy to prove that:

- If $e_i > 0$ then $\Delta(\theta) > 0$ and if $\theta_j > A$ then $\Delta(\theta) > 0$; this implies ti equilibrium does not exist;
- If $e_i > 0$ then $\Delta(\theta) > 0$ and if $\theta_j > A$ then $\Delta(\theta) < 0$; this implies ti equilibrium does not exist;
- If $e_i < 0$ then $\Delta(\theta) < 0$ and $\Delta(\theta) < 0 \forall \theta \in (0, \theta_r)$; def $\theta^*_r = \arg\max\Delta(\theta)$ we get that if $\Delta(\theta^*_r) < 0$ then an endogenous equilibrium does not exist; if $\Delta(\theta^*_r) = 0$ there a unique equilibrium $\theta^*_r \in (0, \theta_r)$.

Propositions 3, 4, and 5 show the effect of an interrelated demand for space. The real option effect is that the first investor's trigger point $\theta_r$ is greater than the strategy $\theta_r$ due to uncertainty and irreversibility (see Propositions 3 and 5b). We can find 1 result in Grenadier (1996). He concludes that in this case it would be optimal for ti to take a preemptive move to reap higher payoff; as a consequence a short in overbidding phenomena might occur. However, introducing asymmetric demands imp Grenadier (1996) will occur only as a limiting case. In fact, Propositions 4, 5a and 5c cases not considered in Grenadier model that reduces his findings to a particular. Specifically, in Propositions 4 and 5a conditions for nonexistence of equilibria arc. This means that there is a strong incentive to follow rather than to lead. Moreover, Prop 5c exploits the range of numerical solutions.
equilibria or a nonexistence of equilibria. It is worth noticing that if \( \theta > \theta_1 \), it is always better to invest.

**Cooperative Solution**

This section analyzes the cooperative solution, in which the agents’ investment trigger points are chosen to maximize the sum of their two value functions. The objective is to provide a benchmark to identify inefficiencies.

Let us examine the case in which investment is sequential. Two trigger points, \( \theta_1 \) and \( \theta_2 \), are chosen to maximize the sum of the leader’s and follower’s value functions, denoted by \( C_{L,F}(\theta) \). Using the same steps as before, it is given by:

\[
C_{L,F}(\theta) = \begin{cases}
\frac{R + B_0 \theta^\alpha + B_1 \theta^\beta}{r} & \text{if } \theta < \theta_2, \\
\frac{-1}{r} \left[ \frac{\eta}{(1 + \frac{1}{2})^\alpha + \frac{1}{4} \theta^\beta - \frac{1}{2}} \right] + 1 + B_0 \theta^\alpha & \text{if } \theta \in [\theta_1, \theta_2], \\
\frac{1}{r} \left[ \frac{\theta}{1 - \frac{1}{4} \theta^\beta} \right] + \beta (B_0 + B_1) \theta^\beta & \text{if } \theta \geq \theta_2.
\end{cases}
\]

where \( B_i, i = 0, 1, 2 \) are constants. The cooperative trigger points are determined by the value matching and smooth pasting conditions at both points. Solving the system, we get the leader trigger and the follower trigger points, \( \theta_1 \) and \( \theta_2 \), respectively, given by:

\[
\theta_1 = \frac{\beta}{\beta - 1} \left[ \frac{\gamma + \frac{1}{2}}{\frac{1}{2} \frac{\gamma}{\frac{1}{2} + \frac{1}{2} \theta^\alpha} + \frac{1}{2} \theta^\beta - \frac{1}{2} \frac{1}{2} \theta^\beta} \right] + \frac{1}{r} - \mu
\]

\[
\theta_2 = \frac{\beta}{\beta - 1} \left[ \frac{\gamma + \frac{1}{2}}{\frac{1}{2} \frac{\gamma}{\frac{1}{2} + \frac{1}{2} \theta^\alpha} + \frac{1}{2} \theta^\beta - \frac{1}{2} \frac{1}{2} \theta^\beta} \right] + \frac{1}{r} - \mu
\]

Equations (29) and (30) identify the trigger values of the leader and the follower, are affected by the interaction effect and externalities. Let us now analyze inefficiency may arise in the non-cooperative equilibria by comparing Equations (29) and (14). Two main results arise:

1. By maximizing the sum of the two value functions, the cooperative leader tries to account for the role of the interaction effect \( \gamma \). In particular, in presence of positive (negative) interaction effect, the non-cooperative leader starts to develop too late (early) with respect to the optimum \( (\theta_1 < \theta_2) \).

2. By maximizing the sum of the two value functions, the cooperative follower only is positively affected by the interaction effect \( \gamma \) and negatively affected \( \epsilon_2 \), but is also negatively affected by the effect that the leader suffers when it develops. In particular, in the presence of negative (positive) externalities, the cooperative follower starts to develop too early (late) with respect to the optimum \( (\theta_2 > \theta_1) \).

Summarizing, the presence of interaction effects and externalities influences the timing (re)development driving the individual choices to no optimal equilibria. In particular, absence of cooperation implies that the agents anticipate or delay the optimal entry if these cases, an important role for the central authority arises and it is possible to implement optimal decisions that move the market into a Pareto-optimum level. The first best for the central authority is to coordinate the individual decisions and reach the optimal time for all developers (e.g., by coordinating and programming a vast redevelopment ex-industrial area, or by changing a mass tourism area into a specialized one). In this case, it is possible to take into account the role of the interaction effects and internalize the externalities that arise when developers operate in the same area. When a direct control developers’ decisions is not possible, it is possible to identify some second and third best policy help the agents to come close to the optimal level. In general, these policies implemented may have direct and indirect effects. Direct policy effects influence developers’ decisions by directly influencing either the costs or returns to development by constraining land use choices and improving the level of public service infrastructure in the area). These policies, referring to the model, act directly on the cost of the developers.

Furthermore, there are the indirect policy effects which influence the externalities of neighbors and so, indirectly, the development cost of the land. These policies, refer to the model, act on \( \eta, \epsilon_1 \), and, indirectly, on the payoffs of the developers.

\[ \text{Note that the optimum is relative to the cooperative benchmark. In this analysis we do not consider consumer welfare aspects of the problem.} \]
Conclusion

Although the real options literature has made a great step towards a better understanding of investment decisions, the contribution of the real option literature to the understanding of the real estate market is still limited. Williams (1991) and Grenadier (1996) were among the first to introduce this methodology in real estate applications. However, in these models, firms are assumed to be identical and products are homogeneous. This symmetric assumption restricts the application of such models only to selected cases.

In this paper, we relax the symmetric hypothesis and analyze the equilibrium strategies of two developers in the real estate market, when demands are asymmetric. We therefore extend the standard real option analysis to a setting where there are general strategic interactions between agents. In particular, we assume demand functions for space are interrelated and may produce either positive or negative externalities. In symmetric demand models, equilibrium strategies either sequential or simultaneous, are driven largely by the action of a comparatively strong leader. This result becomes a special case when we analyze asymmetric demand. We have shown that if the follower’s comparative disadvantage is much weaker than its competition, then the follower will prefer to wait. Moreover, when one firm has a significantly large comparative advantage, the preemptive threat from the rival will be negligible.

Furthermore, we identified the regions of parameter values in which an overbuilding activity with cascading investment or a sluggish activity might occur and showed that overbuilding phenomena as predicted by Grenadier (1996) will occur only as a limiting case and in moving strategies are optimal when the comparative strengths between firms are small; firms will prefer to wait for its competitor to take the first move when the market is volatile.

Finally, we analyze the cooperative solution in which the agents’ investment trigger points are chosen to maximize the sum of their two value functions. Comparing the trigger values of both firms in case of cooperation, we can analyze the inefficiencies that arise in the non-cooperative equilibrium. We find that, without cooperation, the leader does not take into account the presence of the interaction effect ($\eta$) and anticipates (delays) the optimal entry time when $\eta > 0$ ($\eta < 0$) and the follower does not take into account the presence of complementarity to the leader’s value function when both develop ($\epsilon$) and anticipates (delays) the decision to ($\epsilon$) develop when $\epsilon < 0$ ($\epsilon > 0$). In this case, the presence of a central authority that coordinates the development decisions of the agents can avoid an irrational urban planning either internalizing the externalities or considering the role of the interaction effects. Even if a direct control of the developers’ decisions is not possible, it is possible to identify some second-best policies that help the agents to come close to the optimal level by influencing the payoffs directly or by implementing indirect policy effects which influence $\eta$ and $\epsilon$ indirectly.

Several extensions in improving the theoretical framework can be explored in future research. First, we can analyze a microfoundation of the model and analyze its empirical implementation. Second, the duopoly game theoretical framework can be extended to include multi-player dynamic game. Finally, asymmetric equilibrium strategies can be analyzed in an incomplete information framework taking into consideration the asymmetries of firms.

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