Abstract

What degree of tax autonomy should be granted to a taxing authority? Although the policy maker aims at maximizing social welfare, her tax policy may be distorted by the lobbying activity of taxpayers. In this political environment we characterize the conditions under which social welfare can be increased by restricting the set of tax instruments available to the policy maker, i.e. the degree of tax autonomy. We show that full tax autonomy is more costly, in terms both of welfare distortions and lobbying effort, when the lobbies are asymmetric in size, while minimal tax autonomy is more costly when the tax bases are asymmetric across different groups.

Keywords: Tax autonomy, Tax complexity, Special interest groups, Optimal taxation.

JEL codes: D72, H71, H77

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1 Introduction

Actual tax systems are generally complex. In order to achieve a progressive tax burden, personal income taxes are usually non-linear relation, with average tax rates increasing in income. Furthermore, a plethora of tax allowances and tax credits adapt the final tax burden to the source of income (e.g., employee or self-employed), the taxpayer’s household composition (e.g., number of children), the occurrence of particular outlays (e.g., medical expenses, insurance premiums). Although usually characterized by a simple proportional structure, corporate taxes and consumption taxes are also complex. Regarding corporate taxation, reduced tax rates may apply to particular sectors of the economy and, more importantly, rules defining the tax base (i.e., profits) tend to be complex, since most of the items entering the definition of taxable income are subject to evaluation (e.g., investment allowances, inventories). As for indirect taxes (like the RST in the U.S., or the VAT in Europe), they are usually characterized not only by differentiated tax rates on different types of goods and services, but also by exemptions for particular sectors (e.g., health) and by exemption thresholds for small businesses.

Since the pioneering works of Mirrlees [22] and Diamond and Mirrlees [7], a substantial literature on ‘optimal taxation’ has shown that complex taxes can be justified both on efficiency and equity grounds. Given its normative approach, according to which policy makers behave as benevolent social welfare maximizers, this theoretical paradigm implies that an increase of the complexity of tax structures is never harmful for social welfare. In other words, there is no reason, within this type of framework, for limiting or constraining the degree of autonomy enjoyed by the decision maker in shaping the details of the tax structure.

Another strand of the literature, mainly adopting a positive perspective, has focused on the political determinants of tax systems. In particular, Hettich and Winer [15], Dixit and Londregan [9], [10], [11] and [12], and Warshett, Winer and Hettich [29], rely on probabilistic voting theory to explain why taxes take a complex structure. Political parties engaged in electoral competition tend to give a more favorable tax treatment to groups of voters that are characterized by a larger proportion of ‘swing’ voters. In

1The classical textbook account of the optimal taxation paradigm is Atkinson and Stiglitz [1] and, more recently, Myles [24]. Two important and recent contributions on the characterization of the optimal non-linear income tax are Saez [25] and [26].

2Optimal tax theory typically ignores the costs of administering the tax system. ‘Simplicity’ and ‘efficient administration’ are often informally invoked among the merits of ‘flat taxes’ (see Keen, Kim and Varsano [18] for a throughout account of the long lasting debate over flat taxes). Only few theoretical studies account for the fact that a more complex tax structure involves higher administration costs (see, e.g., Yitzhaki [31] and Mayshar [21]).

3Somewhat related is the literature investigating the link between redistribution through public
contrast to optimal tax theory, within this political environment greater complexity of
the tax structure is generally detrimental for social welfare, since political forces do
not necessarily pursue efficiency and equity objectives. However, none of these studies
considers the possibility of restricting the tax autonomy of the policy maker, in order
to assess whether simpler (i.e., more restricted) tax structures lead to better social
outcomes.

In this paper we focus on a concept of autonomy that refers to the possibility of
shaping tax schedules. We examine from a normative perspective whether it is desirable
to restrict the tax autonomy of a policy maker in its administration of a tax that serves
both the financing of a public good and the pursuit of equity objectives, given that
political influence may distort tax policy away from the optimum. In this respect, our
approach is different from that commonly adopted by the swing voters literature (e.g.,
Dixit and Londregan [9], [10]) in that, here, it is not the policy maker that distorts
tax policy in order to please high clout groups. Rather, taxpayers themselves exert
political pressure to affect the decision maker’s tax policy.

In our model, a policy maker is entitled to levy a tax on a tax base that is positively
correlated with the full income (ability to pay) of taxpayers. Taxation is restricted to
be linear but, in principle, it is possible to tax differently groups that have different
observable characteristics (such as source of income, rural or urban residence, etc.).
We then study under what conditions it is optimal to prevent the decision maker from
imposing different tax schedules on different groups.

Our crucial assumption is that tax setting is not driven by pure social welfare max-
imization. Instead, various groups of taxpayers exert political pressure, by means of
lobbying activities, with the aim of gaining a more favorable tax treatment. When lob-

4The policy maker in charge of tax policy can belong either to the central or to the local level
of government. As for the latter, the literature on fiscal federalism has identified several reasons for
limiting the tax autonomy of local governments, but none has yet focused on the one addressed in
this paper. These include efficiency arguments, like tax competition among governments belonging to
the same layer which gives rise to horizontal tax externalities (see, e.g., Wilson [30] and Zodrow and
Mieszkowski [32]), and tax competition between different layers of government sharing the same tax
base which gives rise to vertical tax externalities (Keen [17], Keen and Kotsogiannis [19] and Dahlby
and Wilson [6]). The literature offers also arguments in favor of fiscal autonomy. The classical argument
(see, e.g., Tiebout [27] and Brennan and Buchanan, [4]) is that fiscal autonomy promotes competition
among local governments that hinders their tendency to act as Leviathan revenue maximizers.

5In most political economy applications, lobbying behavior is modeled using the ‘buying influence’
approach of the common agency games developed by Dixit, Grossman and Helpman [8] and Grossman
and Helpman [13], [14]. In this paper, we use a simpler model of lobbying behavior in the spirit of
bying groups successfully influence policy choices, the resulting tax policy is distorted away from the social optimum, provided that the opposing parties do not offset each other in their attempts to gain influence. Even when the battle for political influence does not distort the tax structure, inefficiencies arise because resources are wasted in lobbying. In this setting, restricting the set of the available tax instruments – i.e., restricting tax autonomy – may be welfare improving.

We show that under full tax autonomy (allowing for group-specific tax schedules) the lobbying activity concentrates on group-specific subsidies, hence distorting income distribution *between* groups, while tax rates are unaffected by lobbying and can therefore be used to optimally redistribute income *inside* groups. On the contrary, when tax autonomy is minimal (i.e., when the tax schedule is restricted to be uniform across groups), lobbying distorts all tax instruments, hence affecting income distribution both inside and between groups. In fact we show that, under full tax autonomy, the extent of the distortion away from the social optimum is related to the asymmetry in the size of the different lobbying groups. When tax autonomy is large, relatively small groups have higher incentives to lobby and therefore also have higher size-adjusted political influence than relatively large groups; as observed above, under full tax autonomy the lobbying activity is essentially directed to increase group-specific subsidies. When tax autonomy is restricted, group-specific subsidies become impossible. Redistribution from one group to another, however, is still possible: by setting a uniform tax rate and a uniform subsidy, groups with a tax base higher than average end up subsidizing groups with a tax base lower than average. Thus, under minimal tax autonomy, the lobbying activity is related to the distance of the group’s average tax base from the overall average tax base. In particular, groups with an average tax base equal to the overall average have no incentive to lobby and economies in which average tax bases are similar across lobbying groups end up not suffering much distortion from the lobbying activity.

The basic message of the paper is that restricting tax autonomy is likely to improve social welfare whenever the different lobbying groups are asymmetric in size and the tax base is evenly distributed or, more precisely, when the average values of the group specific tax bases are close to the global average. More tax autonomy is instead desirable when tax bases are asymmetrically distributed and the lobbying groups are of similar size.

The rest of the paper is organized as follows. The model is set up in Section 2, where we specify the types of tax structures available to the policy maker and the process leading to tax setting as a result of the interplay between the policy maker

Becker [2], that can be interpreted as a reduced form of the common agency models.
and the taxpayers organized in lobbying groups. In Sections 3 and 4 we examine the fiscal policies that emerge under the two polar tax environments of Full Tax Autonomy (FTA) and Minimal Tax Autonomy (MTA). Section 5 is devoted to a comparison of the two regimes looking at global welfare. Section 6 concludes. An appendix collects all the proofs.

2 A Model of Tax Policy

We consider a population partitioned into $J$ groups according to some observable characteristics (e.g., source of income, area of residence and so on). Group $j$ has mass $\theta_j \in (0,1)$, with $\sum_{j=1}^J \theta_j = 1$. There is heterogeneity inside groups but the members of each group have to be treated uniformly by the tax policy. The type of each agent is given by a triplet $(\beta, B, \gamma)$, where $\beta$ denotes a measure of the agent’s ability to pay (e.g., income or wealth), $B$ is the tax base on which the policy maker can levy a tax, and $\gamma$ is the unit benefit the agent receives from the public good provided by the government.

We assume that $B$ is an imperfect measure of an agent’s ability to pay, so that in expected terms a taxpayer with a high ability to pay $\beta$ also has a high tax base $B$.

**Assumption 1**

(i) For each group $j$, the distribution $f_j$ is such that $\text{cov}_j(\beta, B) > 0$.
(ii) For the entire population, $\text{cov}(\beta, B) > 0$. (iii) For each group $j$ and for each variable the variance is strictly positive.

The imperfection of $B$ is a common feature of real tax systems, due to the presence of tax-base measurements errors or to tax evasion and tax avoidance phenomena.

We also assume that the public good unit-benefit $\gamma$ is private information. If $G$ units of the public good are produced then an agent of type $(\beta, B, \gamma)$ receives a benefit $\gamma G$, and the cost of public good provision for the community is $G$. We assume that for each group $j$ the average benefit $\tau_j$ of the public good is higher than the constant cost of production.

**Assumption 2** $\tau_j > 1$ for each group $j$.

Notice that Assumption 2 implies that $\sum_{j=1}^J \theta_j \tau_j = \tau > 1$. We do not make assumptions about the relation, in expected terms, between the benefits $\gamma$ and the tax base $B$ or the ability to pay $\beta$, since both positive and negative correlations are possible in practice.\(^6\) As our focus is on tax policy, we take the supply of the public good to be

\(^6\)For instance, in the case of fire and police protection, those with higher property values and incomes have more to benefit from public expenditure. The correlation is instead negative if, for instance, $G$ is interpreted as public schooling and rich households are more likely to send their children to private schools.
given exogenously.\(^7\)

In each group \(j\), the distribution of types is given by the known density \(f_j (\beta, B, \gamma)\) on the set \(\mathbb{R}^3_+\). Denoting with \(E_j [g(x)]\) the expected value of \(g(x)\) for group \(j\), let

\[
\pi_j = E_j [x], \quad \text{var}_j (x) = E_j \left[ (x - \pi_j)^2 \right], \quad \text{cov}_j (x, y) = E_j [xy] - \pi_j \bar{y}_j,
\]

be the average value of \(x\), the variance of \(x\), and the covariance between \(x\) and \(y\) for group \(j\), respectively. For the entire population, let

\[
\pi = \sum_{j=1}^J \theta_j \pi_j, \quad \text{var} (\pi_j) = \sum_{j=1}^J \theta_j (\pi_j - \pi)^2, \quad \text{var} (x) = \sum_{j=1}^J \theta_j \text{var}_j (x) + \text{var} (\pi_j),
\]

\[
\text{cov} (\pi_j, \bar{y}_j) = \sum_{j=1}^J \theta_j \pi_j \bar{y}_j - \pi \bar{y}, \quad \text{cov} (x, y) = \sum_{j=1}^J \theta_j \text{cov}_j (x, y) + \text{cov} (\pi_j, \bar{y}_j).
\]

The policy maker is only allowed to levy group–specific linear taxes on \(B\), with tax rate \(t_j\) and a lump sum subsidy \(S_j\). The lump sum component can be either positive or negative. Denote by \(t = (t_1, \ldots, t_J)\) the vector of tax rates and by \(S = (S_1, \ldots, S_J)\) the vector of subsidies. The net utility for type \((\beta, B, \gamma)\) of group \(j\), at the given level \(G\) of public good provision, is

\[
u (t_j, S_j | \beta, B, \gamma) = \gamma G + \beta - t_j B + S_j. \tag{1}\]

Let

\[
\bar{\pi}_j (t_j, S_j | \beta_j, \bar{B}_j, \tau_j) = E_j [u] = \bar{\gamma}_j G + \bar{\beta}_j - t_j \bar{B}_j + S_j \tag{2}
\]

be the average net utility for members of group \(j\), and

\[
\bar{u} (t, S | \beta, \bar{B}, \tau) = \sum_{j=1}^J \theta_j \bar{\pi}_j = \bar{\gamma} G + \bar{\beta} - \bar{T} + \bar{S} \tag{3}
\]

be the average net welfare for the population as a whole, where

\[
\bar{T} = \sum_{j=1}^J \theta_j t_j \bar{B}_j, \quad \bar{S} = \sum_{j=1}^J \theta_j S_j.
\]

Finally, let

\[
\text{var} (u) = \sum_{j=1}^J \theta_j E_j \left[ (u - \bar{u})^2 \right] = \sum_{j=1}^J \theta_j \text{var}_j (u) + \text{var} (\bar{u}_j) \tag{4}
\]

\(^7\)We discuss the case of endogenous public good supply in Brusco, Colombo and Galmarini [3], where we find that limiting the tax autonomy of a local government may distort the supply of the local public good away from its optimal level. Both under and over provision may occur, depending on whether restricting tax autonomy increases or reduces the social marginal cost in terms of inequality of public good provision.
be the variance of net utilities, which is composed of the within-groups variance, \( \sum_{j=1}^{J} \theta_j \text{var} (u) \), and of the between-groups variance, \( \text{var} (\bar{u}_j) \).

Taxes and subsidies are determined through a lobbying game, the outcome of which can be described in reduced form as the maximization of the objective function

\[
V (t, S) = \sum_{j=1}^{J} q_j \bar{u}_j - r \text{var} (u),
\]

where \( q_j \) is the modified weight of group \( j \) once lobbying activity is taken into account.

We assume that the modified weights are determined as

\[
q_j = h \left( \theta_j, \frac{p_j}{\bar{p}} \right),
\]

where the function \( h \) captures the sensitivity of the policy makers to lobbying, \( p_j \) is the lobbying effort exercised by group \( j \) and \( \bar{p} \) is the average lobbying effort. The lobbying activity by group \( j \) will eventually be made endogenous and it depends on the overall lobbying effort by members of group \( j \). At this stage, however, we take the values \( p_j \), and hence \( q_j \), as given. We make the following assumptions about the function \( h (\theta_j, x) \).

**Assumption 3** The function \( h (\theta_j, x) \) is continuous and differentiable for each \( x \geq 0 \). Furthermore: i) \( \lim_{x \to 0} \frac{\partial h(\theta_j, x)}{\partial x} > 0 \) and \( \frac{\partial h(\theta_j, x)}{\partial x} \) is a continuous and bounded function of \( x \); ii) \( h (\theta_j, 1) = \theta_j \); iii) \( h \left( \theta_j, \frac{p_j}{\bar{p}} \right) \in [0, 1] \) and \( \sum_{j=1}^{J} h \left( \theta_j, \frac{p_j}{\bar{p}} \right) = 1 \) for each pair of vectors \((\theta_1, \ldots, \theta_J)\) and \((p_1, \ldots, p_J)\); iv) \( \frac{\partial^2 h(\theta_j, x)}{\partial x^2} \leq 0 \).

Thus, the weight of group \( j \) increases when the group lobbying effort gets larger compared to the average lobbying effort. When a group exerts an effort equal to the average then its weight is unchanged. The restrictions in iii) just make sure that \( q_j \) remains between 0 and 1 and that \( \sum q_j = 1 \) for each vector of efforts; condition iv) says that the marginal effectiveness of the lobbying effort decreases as the effort gets larger.

Notice that, when written as a function of the vector \((p_1, \ldots, p_J)\), the function \( h \left( \theta_j, \frac{p_j}{\bar{p}} \right) \) is well defined only if \( \bar{p} \neq 0 \), i.e. at least one \( p_j \) is strictly positive. This problem is easily taken care of assuming that whenever \( \bar{p} = 0 \) we have \( h \left( \theta_j, \frac{p_j}{\bar{p}} \right) = \theta_j \).

A simple class of functions that meets all the conditions in Assumption 3 is

\[
h \left( \theta_j, \frac{p_j}{\bar{p}} \right) = \zeta \theta_j \frac{p_j}{\bar{p}} + (1 - \zeta) \theta_j
\]

where \( \zeta \in [0, 1] \) and can be used as a parameter denoting the effectiveness of the lobbying activity (\( \zeta = 0 \) means that lobbying is completely ineffective).
An observation that will be useful later is the following. Since the function $h$ is such that
\[ H(\theta_1, \ldots, \theta_J, p_1, \ldots, p_J) = \sum_{j=1}^{J} h\left(\theta_j, \frac{p_j}{\overline{p}}\right) = 1 \]
for each vector $(p_1, \ldots, p_J)$, it has to be the case that $\frac{\partial H}{\partial p_i} = 0$ for each $i$. Now observe that
\[
\frac{\partial H}{\partial p_i} = \frac{\partial h\left(\theta_i, \frac{\nu}{\overline{p}}\right)}{\partial p_i} \cdot \overline{p} + \sum_{j \neq i} \frac{\partial h\left(\theta_j, \frac{p_j}{\overline{p}}\right)}{\partial p_j} \cdot \frac{p_j}{\overline{p}} = 0.
\]
Using $\frac{\partial h\left(\theta_i, \frac{\nu}{\overline{p}}\right)}{\partial p_i} = \nu - \theta_i \frac{p_i}{\overline{p}}$ and $\frac{\partial h\left(\theta_j, \frac{p_j}{\overline{p}}\right)}{\partial p_j} = -\frac{\theta_j p_j}{\overline{p}}$, after rearrangement we obtain
\[
\frac{\partial h\left(\theta_i, \frac{\nu}{\overline{p}}\right)}{\partial \frac{\nu}{\overline{p}}} = \theta_i \left( \sum_{j=1}^{J} \frac{\partial h\left(\theta_j, \frac{p_j}{\overline{p}}\right)}{\partial \frac{p_j}{\overline{p}}} \frac{p_j}{\overline{p}} \right)
\]
for each $i$. A more convenient way to write this equation is the following. Let $\theta = (\theta_1, \ldots, \theta_J)$ and $p = (p_1, \ldots, p_J)$, and define
\[
\Phi(\theta, p) = \sum_{j=1}^{J} \frac{\partial h\left(\theta_j, \frac{p_j}{\overline{p}}\right)}{\partial \frac{p_j}{\overline{p}}} \frac{p_j}{\overline{p}}.
\]
Then (7) can be written as
\[
\frac{\partial h\left(\theta_i, \frac{\nu}{\overline{p}}\right)}{\partial \frac{\nu}{\overline{p}}} = \theta_i \Phi(\theta, p),
\]
that is, partial derivatives are proportional to the function $\Phi$, with coefficient of proportionality given by $\theta_i$.

A last remark is in order. In our formulation the intensity of lobbying by group $j$ is measured by the ratio $p_j/\overline{p}$. A possible alternative is to use the ratio $\theta_j p_j/\overline{p}$. However this is equivalent, since we can always write a function that depends on $\theta_j$ and $\theta_j p_j/\overline{p}$ as a function that depends on $\theta_j$ and $p_j/\overline{p}$.

Coming back to expression (5), it can be thought as coming from a concave social welfare function, with $r > 0$ determining the level of inequality aversion. However, the weight $q_j$ given to group $j$ is not determined purely by its size but also by its relative political clout, as reflected by the lobbying weight $h\left(\theta_j, \frac{p_j}{\overline{p}}\right)$.

We do not allow for debt, so that the tax and subsidy policy $(t, S)$ has to satisfy the budget constraint
\[
G = T - S.
\]
Since public expenditure is given, the tax and subsidy policy is obtained solving

$$\max_{t,S} \ V(t, S) \quad \text{s.t. (10).} \ (11)$$

We want to analyze the welfare effects of the following two tax regimes.

- **Full Tax Autonomy** (FTA): the policy maker is free to set different tax rates $t_j$ and subsidies $S_j$ for different groups.
- **Minimal Tax Autonomy** (MTA): the policy maker is forced to treat all groups homogeneously. In other words, the decision maker can only choose a pair $(t, S)$ and then set $t_j = t$, $S_j = S$ for each group $j$.

Intermediate cases are also possible, such as those in which tax schedules can vary only within a certain range. In this paper, our goal is to identify the effect of a restriction of tax autonomy on welfare, so we limit our attention to the two extreme cases. A more complete analysis would include a discussion of the exact degree and format of tax autonomy that maximizes social welfare.\(^8\)

### 3 Full Tax Autonomy

We consider a two-stage game. In stage 1, given public expenditure, taxpayers exert political influence on the policy maker. In stage 2, tax policy is determined as outlined above. The model is solved by backward induction.

#### 3.1 Taxes and Subsidies under FTA

At stage 2, taking as given the $p_j$’s, the policy maker solves program (11). The result is given in the following proposition.

**Proposition 1** Under full tax autonomy, the optimal tax rate and subsidy for group $j$ are given by

$$t^*_j = \frac{\text{cov}_j(\beta, B) + G \text{cov}_j(\gamma, B)}{\text{var}_j(B)},$$

$$S^*_j = t^*_j \overline{B}_j - G - (\bar{\pi}_j - \pi) G - (\bar{\beta}_j - \bar{\beta}) + \frac{q_j - \theta_j}{2r \theta_j}. \ (13)$$

---

\(^8\)Immonen, Kanbur, Keen and Tuomala [16] and Viard [28] examine the optimal differentiation of income tax schedules among sub-groups of taxpayers in a classical optimal taxation framework, with endogenous labor supply and a benevolent policy maker that maximizes a social welfare function. The complexity of tax systems has also been investigated by Hettich and Winer [15] and Warskett, Winer, and Hettich [29] in a probabilistic voting framework.
The logic of the result is simple. In our framework taxes have no adverse effect on the income produced. If the planner could observe the type of each agent and establish individual transfers then one simple way to solve program (11) would be to assign lump-sum individual-specific subsidies. In fact, since the planner is inequality-averse, a simple solution is to confiscate entirely the income, finance the production of the public good and then redistribute the remaining tax revenue compensating those who have a lower preference for the public good. For a type \((\gamma, \beta, B)\) this would lead to a personalized marginal tax rate \(t(\gamma, \beta, B) = \beta / B\), for a total tax revenue of \(\bar{\beta}\), and a personalized subsidy

\[
S(\gamma, \beta, B) = \bar{\beta} - G - (\gamma - \bar{\gamma}) G,
\]

where the first term is the total tax revenue, \(G\) is the amount to be financed and the last term compensates those who derive low utility from the public good.

However, the planner does not observe the individual types but only the distribution they are drawn from; i.e., the group \(j\) to which they belong. This leads to higher subsidies for those groups with below-average income and below-average preference for the public good. Since we allow for negative values of \(S_j\), in principle all the subsidies could be financed via lump sum taxes, but a non-zero tax rate \(t_j\) can be used to reduce intra-group inequality. In fact, notice that if \(\gamma, \beta\) and \(B\) were independent, so that \(\text{cov}_j(\beta, B) = \text{cov}_j(\gamma, B) = 0\), then the optimal tax rate would be zero.\(^9\) However, when there is correlation of either \(\gamma\) or \(\beta\) with \(B\) then group-specific tax rates can be used to decrease the variability of utility inside the group. If \(\text{cov}_j(\beta, B) > 0\) and \(\text{cov}_j(\gamma, B) > 0\) then individuals with a higher tax base \(B\) enjoy on average both a higher ability to pay \(\beta\) and a higher utility from the public good \(\gamma\). Setting a positive tax rate and then distributing the tax revenue as a lump sum subsidy decreases the inside-group variance. As observed, the job of \(t_j\) is to redistribute income inside group \(j\), so that the political power of group \(j\) versus other groups is irrelevant.

Considerations of relative political power are instead important in determining the lump-sum subsidy \(S_j^{**}\). In fact, the subsidy to group \(j\) tends to equalize utility across groups, much in the same way as individual lump-sum taxes and subsidies would do. Thus, the subsidy compensates groups with lower than average taste for the public good \((\bar{\gamma}_j < \bar{\gamma})\) and lower than average ability to pay \((\bar{\beta}_j < \bar{\beta})\). The last term in (13), which depends on the relative political influence \(q_j - \theta_j\), is the result of distortionary lobbying activities and it is equal to zero when \(p_j = \bar{p}\) for each \(j\).

\(^9\)This does not mean that group \(j\) does not pay taxes, since \(S_j^{**}\) could be negative.
Substituting $t_j^*$ and $S_j^{**}$ from Proposition 1 into group $j$ average utility (2), we get:

$$
\pi_j (t_j^*, S_j^{**}) = (\gamma - 1) G + \beta + \frac{q_j - \theta_j}{2r\theta_j}.
$$

(14)

This shows that the only source of between-groups inequality is the presence of non-uniform political weights. The average utility at the optimal policy is

$$
\sum_{j=1}^{J} \theta_j \pi_j (t_j^*, S_j^{**}) = (\gamma - 1) G + \beta + \sum_{j=1}^{J} \theta_j \frac{q_j - \theta_j}{2r\theta_j} = (\gamma - 1) G + \beta
$$

(15)

and the between-groups variance

$$
\text{var} (\pi_j (t_j^*, S_j^{**})) = \frac{1}{4r^2} \text{var} \left( \frac{q_j - \theta_j}{\theta_j} \right) = \frac{1}{4r^2} \sum_{j=1}^{J} \frac{(q_j - \theta_j)^2}{\theta_j},
$$

(16)

where we have used $E \left( \frac{q_j - \theta_j}{\theta_j} \right) = 0$ and

$$
\text{var} \left( \frac{q_j - \theta_j}{\theta_j} \right) = \sum_{j=1}^{J} \theta_j \left( \frac{q_j - \theta_j}{\theta_j} - E \left( \frac{q_j - \theta_j}{\theta_j} \right) \right)^2.
$$

This is intuitive. With inequality aversion, a benevolent planner would equalize the average utility of all groups. It is only the presence of differential political power that leads to differences across groups.

### 3.2 Political Lobbying under FTA

The analysis up to now has taken the lobbying efforts $p_i$, and thus the weights $q_i$, as given. We now endogenize those values. We assume that exerting a lobbying effort $p_i$ has a per capita cost for group $i$ members, which takes a quadratic form

$$
c (p_i) = \frac{\psi}{2} p_i^2.
$$

(17)

Observe that, given this cost function, the per-capita lobbying cost is independent of group size. This implies that the cost of lobbying effort is neutral, in the sense that the incentives to lobby of taxpayers belonging to different groups are independent of each group size; i.e., there are not economies of scale in lobbying.

Since the marginal tax rate $t_i^*$ does not depend on $p_i$ we can ignore it in the analysis. Thus, taking as given the lobbying weights of groups other than $i$, the lobby group $i$ sets $p_i$ to solve

$$
\max_{p_i \geq 0} S_i^{**} (p) - \frac{\psi}{2} p_i^2,
$$

(18)
where $S_i^* (p)$ is given by (13). This is equivalent to

$$
\max_{p_i \geq 0} \frac{h \left( \theta_i, \frac{p_i}{\bar{p}} \right)}{2 r_i} - \frac{\psi}{2} p_i^2,
$$

which leads to the condition

$$
2r_i p_i = \frac{\partial h_i \left( \theta_i, \frac{p_i}{\bar{p}} \right)}{\partial \frac{p_i}{\bar{p}}} \bar{p} - \theta_i p_i.
$$

(20)

Using (9), condition (20) can be written as

$$
2r_i p_i = \theta_i \Phi \left( \theta, p \right) \left( \bar{p} - \theta_i p_i \right),
$$

which leads to

$$
p_i = \frac{\Phi \left( \theta, p \right)}{2r_i \bar{p}^2 + \theta_i \Phi \left( \theta, p \right)}.
$$

(21)

At the equilibrium, the condition

$$
\Phi = \sum_{j=1}^{J} \frac{\partial h \left( \theta_j, \frac{\psi}{2r \bar{p}^2 + \theta_j \Phi} \right)}{\partial \frac{p_j}{\bar{p}}} \frac{\Phi}{2r \bar{p}^2 + \theta_j \Phi},
$$

(22)

obtained by substituting $\frac{\psi}{\bar{p}}$ from (21) into (8), has to be satisfied. Furthermore, multiplying both sides (21) by $\frac{p}{\bar{p}}$ and summing over $i$ we obtain

$$
1 = \sum_{i=1}^{J} \frac{\theta_i \Phi}{2r_i \bar{p}^2 + \theta_i \Phi}.
$$

(23)

A solution $(\Phi^{*\text{FTA}}, \bar{p}^{*\text{FTA}})$ of the system (22) – (23) is a candidate equilibrium. The choice $p_i$ of each group is then given by

$$
p_i = \frac{\Phi^{*\text{FTA}} \bar{p}^{*\text{FTA}}}{2r \psi \left( \bar{p}^{*\text{FTA}} \right)^2 + \theta_i \Phi^{*\text{FTA}}}.
$$

(24)

The next proposition establishes the existence of the equilibrium.

**Proposition 2** A solution $(\Phi^{*\text{FTA}}, \bar{p}^{*\text{FTA}})$ to the system (22) – (23) exists. The collection $(p_1^{*\text{FTA}}, \ldots, p_J^{*\text{FTA}})$ such that

$$
p_i^{*\text{FTA}} = \frac{\Phi^{*\text{FTA}} \bar{p}^{*\text{FTA}}}{2r \psi \left( \bar{p}^{*\text{FTA}} \right)^2 + \theta_i \Phi^{*\text{FTA}}}.
$$

is a Nash equilibrium of the lobbying game.
One immediate observation that can be made, looking at equation (24), is that the level of the lobbying activity is decreasing in the size of the group \( i \). This is a consequence of our assumption that the effectiveness of effort by group \( i \) depends on the ratio \( p_i/p \). When a group increases \( p_i \) it also increases \( p \), with the latter increase being larger the larger is \( \theta_i \), the size of the group. This implies that lobbying is less productive for large groups.

A second observation is that equation (24) implies that the asymmetry in size is the only determinant of the difference in lobbying activity. In fact, when all the groups are of the same size, i.e. \( \theta_i = 1/J \), then the lobbying activity is identical for all groups and therefore we have \( h(\theta_i, \frac{p_i}{p}) = h(\theta_i, 1) = \theta_i \) for each group \( i \). This implies that social welfare is maximized with the correct weights, and the only costs from the lobbying activity are the direct ones. Thus, under full tax autonomy, deviations from the socially optimal decisions due to lobbying are possible only when the groups have asymmetric size.

4 Minimal Tax Autonomy

In this section, we consider the case in which a ‘no discrimination among groups’ rule is imposed, thus restricting tax autonomy. This implies that tax rates and subsidies have to be the same across groups, i.e. \( t_j = t \) and \( S_j = S \) for each \( j \).

4.1 Taxes and Subsidies under MTA

Under the ‘no discrimination’ rule, the utility of type \( (\beta, B, \gamma) \) is independent of the group \( j \) to whom she belongs. Since

\[
 u - \bar{u} = (\gamma - \bar{\gamma}) G + (\beta - \bar{\beta}) - t (B - \bar{B}),
\]

the variance of the utilities is equal to

\[
 \text{var} (u) = G^2 \text{var} (\gamma) + \text{var} (\beta) + t^2 \text{var} (B) + 2 (G \text{cov} (\gamma, \beta) - tG \text{cov} (\gamma, B) - tcov (\beta, B)) .
\]

Thus, the optimal tax problem solved by the policy maker in stage 2 can be written as

\[
 \max_{t,S} \sum_{j=1}^{J} q_j (\gamma_j G + \beta_j tB_j + S) - r \text{var} (u) \quad \text{s.t.} \quad G = t\bar{B} - S.
\]

Define

\[
 \text{cov} (\theta_j - q_j, \bar{B}_j) = \sum_{j=1}^{J} (\theta_j - q_j) (\bar{B}_j - \bar{B}) = -\sum_{j=1}^{J} q_j (\bar{B}_j - \bar{B}).
\]

Then, solving problem (26) we obtain the following result.
Proposition 3 Under minimal tax autonomy, the optimal tax and subsidy for all groups are given by

\[ t^{**}(p) = \frac{\text{cov}(\beta, B) + G \text{cov}(\gamma, B)}{\text{var}(B)} + \frac{\text{cov}(\theta_j - q_j, B_j)}{2r\text{var}(B)}, \]  

(27)

and

\[ S^{**}(p) = t^{**}(p,G)\bar{B} - G. \]  

(28)

To understand Proposition 3, consider first the case in which the decision is not distorted by lobbying, so that \( \text{cov}(\theta_j - q_j, B_j) = 0 \). In this case, the tax rate (27) is similar to the one we found for the case of full tax autonomy. Of course, since the tax rate is the same across groups, we have to use the distribution for the overall population \( f = \sum_{j=1}^{J} \theta_j f_j \) rather than the group-specific distributions \( f_j \). Other than that, the principles behind the determination of the tax rate are the same. Notice that if the tax base \( B \) is not correlated to either \( \beta \) or \( \gamma \) then the optimal tax rate is \( t = 0 \). In turn, this implies that \( S = -G \), so that the public good is financed through a lump-sum tax equal for all citizens. Instead, when \( \text{cov}(\beta, B) > 0 \), the planner sets a positive tax rate since those who end up paying the tax are also on average the ones who have a higher income \( \beta \). Thus, a positive tax rate reduces inequality. A similar reasoning applies when \( \text{cov}(\gamma, B) > 0 \).

When political distortion is added, so that \( \text{cov}(\theta_j - q_j, B_j) \neq 0 \), the tax rate changes. This is an important difference with respect to the case of full tax autonomy, as in that case lobbying only influences subsidies. If groups with below-average local tax base have higher political influence, so that \( \text{cov}(\theta_j - q_j, B_j) > 0 \), then the marginal tax rate tends to be higher than in the absence of political influence. The reason is that in this case the tendency of the government to distribute taxes from rich to poor groups is strengthened by the additional weight which is placed on poor groups. More in general, notice that

\[ \frac{\partial t^{**}(p)}{\partial q_j} = \frac{\bar{B} - B_j}{2r\text{var}(B)}, \]

so that an increase in the political weight \( q_j \) in principle affects the marginal tax rate in a different way depending on whether the average tax base of group \( j \) is below or above the average tax base for the whole population. If the average taxpayer of group \( j \) is ‘poor’, so that \( B_j < \bar{B} \), then an increase in her political influence determines an increase in the progressiveness of the tax schedule, by increasing both the marginal tax rate and the lump sum subsidy. The opposite effect holds for a group in which the average taxpayer has a higher-than-average tax base (i.e., \( B_j > \bar{B} \)).
The previous reasoning however is only meant to be illustrative, since $q_j$ cannot change in isolation (remember that the sum of the $q_j$'s has to be one). If we look at the effect of a change in the lobbying effort $p_j$ we have the following result.

**Proposition 4** The derivative of $t^*(p)$ with respect to $p_j$ is

$$\frac{\partial t^*(p)}{\partial p_j} = \frac{\theta_j \Phi(\theta, p)}{2pr \text{var}(B)} \left( \sum_{i=1}^{J} \frac{\theta_i p_i}{\bar{p}} \bar{B}_i - \bar{B}_j \right). \quad (29)$$

Notice that an equivalent way to write $\frac{\partial t^*(p)}{\partial p_j}$ is

$$\frac{\partial t^*(p)}{\partial p_j} = \frac{\theta_j \Phi(\theta, p)}{2pr \text{var}(B)} \left( \sum_{i \neq j} \frac{\theta_i p_i}{\bar{p}} (\bar{B}_i - \bar{B}_j) \right). \quad (30)$$

The proposition then implies that

$$\frac{\partial t^*(p)}{\partial p_j} \geq 0 \iff \sum_{i=1}^{J} \frac{\theta_i p_i}{\bar{p}} \bar{B}_i \geq \bar{B}_j \iff \sum_{i \neq j} \theta_i p_i (\bar{B}_i - \bar{B}_j) \geq 0, \quad (31)$$

i.e., an increase in lobbying by group $j$ increases the tax rate if and only if its average tax base $\bar{B}_j$ is lower than the weighted sum of average tax bases $\sum_{i=1}^{J} \frac{\theta_i p_i}{\bar{p}} \bar{B}_i$, where the weights of the groups depend on their contribution to $\bar{p}$.

In fact, the last implication of (31) tells us that the sign of $\frac{\partial t^*(p)}{\partial p_j}$ does *not* depend on $p_j$. In other words, the lobby will always push the tax rate in the same direction and the direction depends on the lobbying activity of the other groups. Changing $p_j$ can change the size of $\frac{\partial t^*(p)}{\partial p_j}$ but not the sign. This result will be very useful in characterizing the equilibrium of the lobbying game.

### 4.2 Political Lobbying under MTA

In stage 1, given the lobbying effort exerted by other groups, the lobby group $j$ chooses $p_j$ to maximize her expected utility. The problem can be written as

$$\max_{p_j \geq 0} S^*(p_j, p_{-j}) - t^*(p_j, p_{-j}) \bar{B}_j - \frac{\psi}{2} p_j^2,$$

and, using (28), we obtain

$$\max_{p_j \geq 0} t^*(p_j, p_{-j}) (\bar{B} - \bar{B}_j) - \frac{\psi}{2} p_j^2. \quad (32)$$

It is immediate to notice that whenever $\bar{B}_j = \bar{B}$ then the optimal lobbying level must be zero. In fact, the optimal amount of lobbying can be strictly positive only if $\frac{\partial t^*(p)}{\partial p_j}$
has the same sign as \( (\overline{B} - \overline{B}_j) \). When this does not happen, the optimal amount of lobbying is zero. More in general, denoting with \( \lambda_j \) the Lagrange multiplier associated to the non-negativity constraint on \( p_j \), the first order conditions are

\[
\frac{\partial t^* (p_j, p_{-j})}{\partial p_j}(\overline{B} - \overline{B}_j) - \psi p_j + \lambda_j = 0,
\]

\[
\lambda_j p_j = 0 \quad \lambda_j \geq 0 \quad p_j \geq 0,
\]

which can be summarized by

\[
p_j = \frac{1}{\psi} \max \left\{ \frac{\partial t^* (p_j, p_{-j})}{\partial p_j}(\overline{B} - \overline{B}_j), 0 \right\}.
\]

(33)

Among the active groups, those with an average tax base \( \overline{B}_j \) lower than the global average \( \overline{B} \) push the tax rate upward, while the opposite holds for the groups with an average tax base higher than the global average.

By substituting for \( t^*/\partial p_j \) from (30) into (33), an equilibrium can be found solving the following system of \( J + 1 \) equations:

\[
p_j = \frac{\theta_j \Phi}{2 \overline{p} \psi \text{var} (B)} \max \left\{ \left( \overline{B} - \overline{B}_j \right) \left( \sum_{i \neq j} \frac{\theta_i p_i}{\overline{p}} \left( \overline{B}_i - \overline{B}_j \right) \right), 0 \right\}
\]

\[
\Phi = \sum_{j=1}^J \frac{\partial h \left( \theta_i, \frac{p_i}{\overline{p}} \right)}{\partial \frac{p_i}{\overline{p}}} \frac{p_i}{\overline{p}}
\]

defining the equilibrium. One important observation is that the level of public expenditure \( G \) does not appear in the system. Thus, the level of lobbying is independent of the level of \( G \).

**Proposition 5** A Nash equilibrium \((p_1^\text{MTA}, \ldots, p_J^\text{MTA})\) exists. For each group \( j \), the equilibrium effort \( p_j^\text{MTA} \) satisfies

\[
p_j^\text{MTA} = \frac{\theta_j \Phi (\theta, p^\text{MTA})}{2 \overline{p}^\text{MTA} \psi \text{var} (B)} \max \left\{ \left( \overline{B} - \overline{B}_j \right) \left( \sum_{i=1}^J \frac{\theta_i p_i^\text{MTA}}{\overline{p}^\text{MTA}} \overline{B}_i - \overline{B}_j \right), 0 \right\}
\]

(34)

One conclusion that can be reached from expression (33) is that if \( \overline{B}_j > \overline{B} \) and group \( j \) is active (i.e., \( p_j > 0 \)) then it must be the case that each group with an index higher than \( j \) is also active. The reason is that, from expression (29), \( \frac{\partial t^* (p)}{\partial p_j} \leq 0 \) implies \( \frac{\partial t^* (p)}{\partial p_i} < 0 \) for each \( i > j \). Similarly, if \( \overline{B}_j < \overline{B} \) and group \( j \) is active then each group with \( i < j \) must also be active. More in general, notice that for all active groups the amount of lobbying is increasing in the distance \( |\overline{B} - \overline{B}_j| \). Clearly, given that under
a uniform linear tax a balanced-budget increase in the marginal tax rate redistributes from individuals with above-average tax base to individuals with below-average tax base, the incentive to exert effort for political influence becomes larger the greater is the distance of the group’s average tax base from the population average tax base. Thus, in equilibrium, ‘poor’ groups actively lobby for higher taxes, ‘rich’ groups lobby for lower taxes and ‘middle class’ groups don’t lobby much.

It is also obvious that in the case $\bar{B}_i = \bar{B}$ for each $i$ the lobbying activity is zero for each group. In the more interesting case in which $\bar{B}_1 \neq \bar{B}_J$, we have $\bar{B}_1 < \bar{B} < \bar{B}_J$ and $\bar{B}_1 < \sum_{i=1}^J \theta_i p_i \bar{B}_i < \bar{B}_J$ for each vector $(p_1, \ldots, p_J)$. Thus, there are always at least an active group with $\bar{B}_i < \bar{B}$ and one active group with $\bar{B}_J > \bar{B}$. We conclude that when $\bar{B}_1 \neq \bar{B}_J$, in equilibrium there will be an index $i^*$ and an index $j^* > i^*$ such that all groups with $i \leq i^*$ and $i \geq j^*$ will be active, while groups with $i^* < i < j^*$ will set $p_i = 0$ (notice that $j^* = i^* + 1$ is possible, in which case all groups are active).

Under MTA, both the within-groups variance and the between-groups variance may be distorted by political influence. A group’s political influence is related to the distance of the group’s average tax base from the mean tax base of the entire population. Notice also that a high level of political influence is not necessarily associated to a more favorable tax treatment, since there may be an opposing group (on the other side of the average tax base) that succeeds in pulling the tax rate in the opposite direction.

5 Comparing Tax Regimes

As it is clear from the analysis above, the two tax regimes yield quite different outcomes. In this section we discuss the impact of the choice of the tax regime on social welfare.

When lobbying is completely ineffective ($h(\theta_j, x) = \theta_j$ for each $\theta_j$ and $x$) then it is clear that no lobbying effort is undertaken under both tax regimes, so that the policy maker maximizes social welfare using the ‘correct’ weights $(\theta_1, \ldots, \theta_J)$. Concerning tax policy, for any given level of public good supply, FTA does weakly better than MTA simply because there are more instruments available. The computations above show that in fact under FTA the additional flexibility is exploited, thus social welfare is strictly higher under FTA than under MTA.

When is MTA going to be better than FTA? Computations are complex, as they involve both direct and indirect effects, but some observations can be made when the direct cost of lobbying is small compared to the welfare cost induced by distorted social decisions.

The important observation here is that the equilibrium lobbying weights $q_j$ depend on the distances $|\bar{B}_j - \bar{B}|$ under MTA, while they do not under FTA. The best case
for MTA occurs when $B_j = B$, so that equilibrium lobbying is zero and the cost of distortionary lobbying is zero. On the other hand, the distortion under FTA is larger when the sizes of the lobbying groups are asymmetric, while it is zero when the groups have the same size. The conclusion is therefore that restrictions in tax autonomy are appropriate for situations in which lobbying groups are very different in size (which increases the cost of FTA), but they have similar tax bases (which decreases the cost of MTA).

6 Concluding remarks

Real-world tax systems are typically characterized by a relevant degree of complexity. A very large literature on optimal taxation has shown that the design of complex tax schedules by benevolent social welfare maximizing policy makers can be justified both on efficiency and on equity grounds. Therefore, in a normative perspective, there are no reasons to constrain the degree of autonomy enjoyed by decision makers in shaping the tax structure. This notwithstanding, in a more positive perspective, political processes may have a large impact on the design of tax policies. For instance, complex tax systems may emerge either because of decision makers’ willingness to provide favorable tax systems to ‘swing’ voters, or because of the lobbying effort of organized interest groups. Within this political environment, an increase in the complexity of the tax structure may well be detrimental to welfare. Therefore, limiting the extent of autonomy that decision makers enjoy in designing tax policies may lead, under certain conditions, to an increase in social welfare.

The investigation of these conditions is the main contribution of this paper, which focuses on the welfare implications of tax autonomy when governments are subject to the pressure of interest groups. As argued above, while tax autonomy may help decision makers to design tax policies that allow a better fit with the needs of their constituencies, it may also increase the likelihood that they end up being captured by lobbies, hence distorting fiscal policies. In a simple theoretical framework, we show that restricting the degree of tax autonomy may be welfare improving. In particular, our analysis shows that restricting tax autonomy is more likely to be beneficial when the different pressure groups have similar average tax bases and when the groups are asymmetric in size.

Analytical tractability induced us to make some simplifying assumptions raising issues that need to be addressed in future research. A few are worth mentioning, although for the most part they would greatly complicate the structure of the model without undermining its main conclusions. First, public good supply decisions have
been taken as given, so that the size of the government budget is fixed exogenously. It is left for future research to address the implications for tax autonomy of the interplay between tax policy and public good provision, and in particular of the implications of lobbying both on taxes and on the public good.

A second and conceptually more demanding extension would be to explicitly model the extensive form of the lobbying game between the taxpayers and the policy makers, which is taken as a reduced form in the present version of the paper. This may provide a better understanding of the incentives to lobby under different tax regimes, which in turn may be important in designing the optimal structure of the tax system.
Appendix

Proof of Proposition 1. Using (1), (2), (3) and (10), observe that
\[ u - \bar{u}_j = (\gamma - \tau_j) G + (\beta - \bar{\beta}_j) - t_j (B - \bar{B}_j), \]
\[ \bar{u}_j - \bar{u} = G + (\bar{\tau}_j - \bar{\gamma}) G + (\bar{\beta}_j - \bar{\beta}) - (t_j \bar{B}_j - S_j), \]
so that the within-groups variance can be written as
\[ \sum_{j=1}^{J} \theta_j \text{var}_j (u) = \sum_{j=1}^{J} \theta_j \left( G^2 \text{var}_j (\gamma) + \text{var}_j (\beta) + t_j^2 \text{var}_j (B) \right) + \]
\[ + 2 \sum_{j=1}^{J} \theta_j \left( G \text{cov}_j (\gamma, \beta) - t_j G \text{cov}_j (\gamma, B) - t_j \text{cov}_j (\beta, B) \right), \quad (35) \]
while the between-groups variance is
\[ \text{var} (\bar{u}_j) = \sum_{j=1}^{J} \theta_j \left( G + (\bar{\tau}_j - \bar{\gamma}) G + (\bar{\beta}_j - \bar{\beta}) - (t_j \bar{B}_j - S_j) \right)^2. \quad (36) \]
The objective function is concave and the constraint set is convex. Thus, the solution can be found looking at the stationary points of the Lagrangian
\[ L = \sum_{j=1}^{J} q_j \bar{u}_j - r \sum_{j=1}^{J} \theta_j \text{var}_j (u) - r \text{var} (\bar{u}_j) - \mu (G - \bar{T} + \bar{S}). \quad (37) \]
The first order condition with respect to \( S_j \) is
\[ \frac{\partial L}{\partial S_j} \equiv q_j - 2r \theta_j \left( G + (\bar{\tau}_j - \bar{\gamma}) G + (\bar{\beta}_j - \bar{\beta}) - (t_j \bar{B}_j - S_j) \right) - \mu \theta_j = 0, \quad (38) \]
and the first order condition with respect to \( t_j \) is
\[ \frac{\partial L}{\partial t_j} \equiv - \frac{\partial L}{\partial S_j} \bar{B}_j - 2r \theta_j (t_j \text{var}_j (B) - G \text{cov}_j (\gamma, B) - \text{cov}_j (\beta, B)) = 0. \quad (39) \]
Substituting for \( \frac{\partial L}{\partial S_j} = 0 \) into (39), we obtain the expression for \( t_j^* \) given in the proposition. Summing the first order conditions (38) over \( j \) and using the budget constraint we get \( \mu = 1 \). Substituting into (38) and solving for \( S_j \), we obtain the formula given in the proposition.

Proof of Proposition 2. For each \( \Phi > 0 \) there is a unique solution \( \bar{\Phi} (\Phi) \) of equation (23). The solution is continuous, increasing and it satisfies \( \lim_{\Phi \to 0} \bar{\Phi} (\Phi) = 0 \). Consider the equation
\[ \sum_{j=1}^{J} \left( \frac{\partial h \left( \theta_j, \frac{\Phi}{2r \psi (\bar{\Phi})^2 + \theta_j \Phi} \right)}{\partial \Phi} \right) = 1 \quad (40) \]
By Assumption 3, part (i), the LHS is strictly positive for each $\Phi > 0$. Since $\frac{\partial h(\theta, x)}{\partial x}$ is bounded, the LHS goes to zero as $\Phi$ goes to $+\infty$. Since $\lim_{x \to 0} \frac{\partial h(\theta, x)}{\partial x} > 0$, the LHS goes to $+\infty$ as $\Phi$ goes to zero. Continuity then implies that equation (40) has a solution. Let $\Phi^{*FTA}$ be the lowest solution. The pair $(\Phi^{*FTA}, \mathbf{p}(\Phi^{*FTA}))$ is then a solution of the system (22) – (23).

We now show that the solution found is in fact an equilibrium. Consider player $i$ and take the lobbying weights $p_j^{*FTA}$, with $j \neq i$, as given. We want to show that $p_i^{*FTA}$ is a maximizer of problem (19). Thus, suppose that lobby groups other than $i$ set 

$$p_j^{*FTA} = \frac{\Phi^{*FTA} \mathbf{p}^{*FTA}}{2r\psi (\mathbf{p}^{*FTA})^2 + \theta_j \Phi^{*FTA}}.$$ 

Let

$$U_i (\theta_i, p_i, p_{-i}^{*FTA}) = \frac{h\left(\theta_i, \theta_ip_i + \sum_{j \neq i} p_j^{*FTA}\right)}{2r\theta_i (\theta_ip_i + \sum_{j \neq i} \theta_j p_j^{*FTA})^2} - \frac{\psi}{2} p_i^2$$ 

be the objective function of group $i$. The first derivative can be written as

$$\frac{dU_i}{dp_i} = \frac{\partial h(\theta, x)}{\partial x} \left(\frac{\sum_{j \neq i} \theta_j p_j^{*FTA}}{2r\theta_i (\theta_ip_i + \sum_{j \neq i} \theta_j p_j^{*FTA})^2} - \psi p_i\right)$$

and, by construction, it is zero at $p_i^{*FTA} = \frac{\Phi^{*FTA} \mathbf{p}^{*FTA}}{2r\psi (\mathbf{p}^{*FTA})^2 + \theta_i \Phi^{*FTA}}$. The second derivative is

$$\frac{d^2U}{d^2p_i} = \frac{1}{2r\theta_i} \left(\frac{\partial^2 h(\theta, x)}{\partial^2 x} \left(\frac{\sum_{j \neq i} \theta_j p_j^{*FTA}}{(\theta_ip_i + \sum_{j \neq i} \theta_j p_j^{*FTA})^2}\right)^2 - \psi \right),$$

which is strictly negative because, by Assumption 3, $\frac{\partial^2 h(\theta, x)}{\partial^2 x} \leq 0$ and $\frac{\partial h(\theta, x)}{\partial x} > 0$.

**Proof of Proposition 3.** Let $\mu$ be the Lagrange multiplier for the budget constraint. The first order condition with respect to $S$ yields $\mu = 1$. It follows that the first order condition with respect to $t$ is

$$\sum_{j=1}^{J} (\theta_j - q_j) B_{ij} - 2r (t \text{ var} (B) - G \text{ cov} (\gamma, B) - \text{ cov} (\beta, B)) = 0. \quad (41)$$
Solving (41) for \( t \) we obtain the value of \( t^* \) given in the proposition. Substituting into the first order condition for \( S \) we obtain the formula for the optimal subsidy.

**Proof of Proposition 4.** Differentiating the expression of \( t^*(p, G) \) with respect to \( p_j \) we obtain

\[
\frac{\partial t^*(p)}{\partial p_j} = \frac{(B - B_j)}{2r \text{var} (B)} \frac{\partial h \left( \theta_j, \frac{p_j}{\bar{p}} \right)}{\partial \frac{p_j}{\bar{p}}} - \sum_{i \neq j} \frac{(B_i - B)}{2r \text{var} (B)} \frac{\partial h \left( \theta_i, \frac{p_i}{\bar{p}} \right)}{\partial \frac{p_i}{\bar{p}}} \frac{p_i}{\bar{p}} =
\]

\[
= \frac{(B - B_j)}{2r \text{var} (B)} \frac{\partial h \left( \theta_j, \frac{p_j}{\bar{p}} \right)}{\partial \frac{p_j}{\bar{p}}} \frac{p - \theta_j p_j}{p^2} + \theta_j \sum_{i \neq j} \frac{(B_i - B)}{2r \text{var} (B)} \frac{\partial h \left( \theta_i, \frac{p_i}{\bar{p}} \right)}{\partial \frac{p_i}{\bar{p}}} \frac{p_i}{\bar{p}^2} =
\]

\[
= \frac{(B - B_j)}{2r \text{var} (B)} \frac{\partial h \left( \theta_j, \frac{p_j}{\bar{p}} \right)}{\partial \frac{p_j}{\bar{p}}} \frac{p}{p^2} + \theta_j \sum_{i = 1}^{J} \frac{(B_i - B)}{2r \text{var} (B)} \frac{\partial h \left( \theta_i, \frac{p_i}{\bar{p}} \right)}{\partial \frac{p_i}{\bar{p}}} \frac{p_i}{\bar{p}^2}
\]

and, using

\[
\frac{\partial h \left( \theta_i, \frac{p_i}{\bar{p}} \right)}{\partial \frac{p_i}{\bar{p}}} = \theta_i \left( \sum_{j = 1}^{J} \frac{\partial h \left( \theta_i, \frac{p_i}{\bar{p}} \right) p_j}{\partial \frac{p_j}{\bar{p}}} \right) = \theta_i \Phi \left( \theta, p \right),
\]

we can write

\[
\frac{\partial t^*(p)}{\partial p_j} = \frac{\theta_j \phi \left( \theta, p \right)}{2r \text{var} (B) \bar{p}} \left( \sum_{i = 1}^{J} \theta_i p_i (B_i - B_j) \right),
\]

as stated in the proposition.

**Proof of Proposition 5.** Consider the system of \( J \) equations:

\[
\frac{p_j}{\bar{p}} = \frac{\theta_j \phi \left( \theta, p \right)}{2r \text{var} (B) \bar{p}} \frac{1}{\sum_{i = 1}^{J} \theta_i p_i (B_i - B_j)}, \quad j = 1, \ldots, J.
\]

Multiplying by \( \theta_j \) and summing over \( j \) we have \( \sum_{j = 1}^{J} \theta_j \frac{p_j}{\bar{p}} = 1 \). Thus:

\[
2r \text{var} (B) \bar{p}^2 = \Phi \sum_{j = 1}^{J} \theta_j^2 \max \left\{ (B - B_j) \left( \sum_{i = 1}^{J} \theta_i \frac{p_i}{\bar{p}} (B_i - B_j) \right), 0 \right\}.
\]

Let \( x_i = \frac{p_i}{\bar{p}} \) and consider the system of equations

\[
x_j = \frac{\theta_j \max \left\{ (B - B_j) \left( \sum_{i = 1}^{J} \theta_i x_i (B_i - B_j) \right), 0 \right\}}{\sum_{j = 1}^{J} \theta_j^2 \max \left\{ (B - B_j) \left( \sum_{i = 1}^{J} \theta_i x_i (B_i - B_j) \right), 0 \right\}}, \quad j = 1, \ldots, J.
\]
This is a continuous mapping from $\prod_{i=1}^{J} [0, \frac{1}{\theta_i}]$ into itself and it therefore has a fixed point. Furthermore, the fixed point $(x_{1}^{\text{MTA}}, \ldots, x_{J}^{\text{MTA}})$ is such that $\sum_{j=1}^{J} x_{j}^{\text{MTA}} > 0$.

Define

$$\Phi^{\text{MTA}} = \sum_{j=1}^{J} \frac{\partial h(\theta, x_{j}^{\text{MTA}})}{\partial x} x_{j}^{\text{MTA}},$$

and

$$p^{\text{MTA}} = \left( \frac{\Phi^{\text{MTA}} \sum_{j=1}^{J} \theta_{j}^2 \max \left\{ \left( \sum_{i=1}^{J} \theta_{i} x_{i}^{\text{MTA}} (\bar{B}_i - \bar{B}_j) \right) \left( \bar{B} - \bar{B}_j \right), 0 \right\} }{2 \psi \var(\bar{B})} \right)^{\frac{1}{2}}$$

We claim that the collection of lobbying efforts $(p_{1}^{\text{MTA}}, \ldots, p_{J}^{\text{MTA}})$ such that

$$\frac{p_{j}^{\text{MTA}}}{p^{\text{MTA}}} = \frac{\theta_{j} \Phi^{\text{MTA}}}{2 \left( \frac{p^{\text{MTA}}}{p^{\text{MTA}}} \right)^{2} \psi \var(\bar{B})} \max \left\{ \left( \sum_{i \neq j} \theta_{i} x_{i}^{\text{MTA}} (\bar{B}_i - \bar{B}_j) \right) \left( \bar{B} - \bar{B}_j \right), 0 \right\}$$

is a Nash equilibrium. When $(\bar{B} - \bar{B}_j) \left( \sum_{i \neq j} \theta_{i} x_{i}^{\text{MTA}} (\bar{B}_i - \bar{B}_j) \right) \leq 0$ then $p_{j}^{\text{MTA}} = 0$ is clearly optimal. Otherwise, the sign of the second derivatives depends on the sign of $\frac{\partial}{\partial p_{j}} \left( \theta_{j} \Phi(\theta, p) / p^{2} \right)$. Since $\theta_{j} \Phi(\theta, p) = \frac{\partial h(\theta_{j}, p)}{\partial p_{j}}$ we have

$$\theta_{j} \frac{\partial \Phi(\theta, p)}{\partial p_{j}} = \frac{\partial^{2} h(\theta_{j}, p)}{\partial^{2} p_{j}} \frac{\sum_{i \neq j} \theta_{i} p_{i}}{p^{2}} < 0.$$

At last, observe that

$$\frac{\partial}{\partial p_{j}} \left( \frac{\theta_{j} \Phi(\theta, p)}{p^{2}} \right) = \frac{p^{2} \theta_{j} \frac{\partial h(\theta, p)}{\partial p_{j}} - 2 \theta_{j}^{2} p \Phi(\theta, p)}{p^{2}} < 0,$$

which implies that whenever $(\bar{B} - \bar{B}_j) \left( \sum_{i \neq j} \theta_{i} p_{i} (\bar{B}_i - \bar{B}_j) \right) > 0$ the objective function is concave in $p_{j}$ and the point $p_{j}^{\text{MTA}}$ at which the derivative is zero is a global maximizer.
References


