Experimental observation of the X-shaped near field spatio-temporal correlation of ultra-broadband twin beams

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In this work we present the experimental observation of the non-factorable near field spatio-temporal correlation of ultra-broadband twin beams generated by parametric down conversion (PDC), in an interferometric-type experiment using sum frequency generation, where both the temporal and spatial degrees of freedom of PDC light are controlled with very high resolution. The revealed X-structure of the correlation is in accordance with the predictions of the theory.

Spatial and temporal aspects of non linear optical phenomena are often considered separately, but when dealing with broadband phenomena where the presence of angular dispersion plays an important role, a correct description requires accounting for space-time coupling. This has become clear in many fields of nonlinear optics, where recent studies have revealed a rich variety of phenomena that cannot be predicted within the traditional separation between space and time. This concerns for instance the study and generation of spatiotemporal solitons [1], of light bullets in arrays of waveguides [2], or the generation of X-waves [3–5].

The PDC process, now used for several quantum information and communication schemes, has been studied for more than forty years in different contexts but usually considering separately space and time. In the first experimental work on PDC, Burnham and Weinberg [6] showed that the down-converted signal and idler photons appear simultaneously within the resolving time of the detectors and the associated electronics. Their temporal correlations were studied in the pioneering works of Mandel and coworkers (see e.g. [7]), giving rise to the term "twin photons", since the temporal correlation indicated that pairs of photons are born simultaneously. In general, much attention has been devoted to studying the temporal interference effects involving the signal and idler photons, typically manipulated in the far-field of the nonlinear crystal (see e.g. [8, 9]). On the other hand, transverse spatial effects observed in PDC photon pairs, such as the presence of a purely quantum spatial correlation both in the near and far-field from the source [12–16], or in the angular momentum and angular position (see e.g [17]), have provided interesting testing ground for studies of non-classical states of light, and novel schemes for quantum imaging and quantum information [10, 11].

The question "Do the signal and idler photons exit at the same time and at the same position from the output face of the nonlinear crystal?" would typically receive the answer "yes" or "yes, within some spatial and temporal uncertainty" also as a consequence of the above described separable spatio-temporal picture. In fact, in quadratic processes such as PDC, a space-time coupling originates directly from phase-matching (PM), which couples angles and frequencies through angular dispersion. For instance, the hyperbolic geometry of the PDC gain curve in the transverse wave vector-frequency domain implies, as shown in [18, 19], a non factorable geometry of the spatio-temporal coherence properties of the generated radiation. Recently, theoretical investigations [20–22] have revealed that the same hyperbolic geometry of PM is also at the origin of the non-factorable PDC biphoton correlation (also called X-entanglement), which presents a X-shape with a central narrow peak and skewed arms in the space-time domain.

It seems therefore that it is only when we use a model where space and time are simultaneously taken into account that we can correctly answer the raised question. More precisely the answer should be "yes at the crystal output face the two photons can be found at the same place and at the same time within a narrow uncertainty; but one can also find, even if with lower probability, the twin photons relatively displaced in space by a certain amount, provided they arrive with a proportional time delay". Nevertheless, a direct verification of this X-shaped near-field correlation has to date never been made, as experiments on twin beam or twin photons have always focussed, as mentioned before, on their either spatial or temporal properties.

In this paper we present -to the best of our knowledge-the first experimental observation of the X-type [30] non-factorable correlation, in space and time, of broadband twin beams. The challenge of the experiment, performed in the high gain regime, is the ability to independently control the spatial and the temporal degrees of freedom of PDC light with a resolution in the micrometer and femtosecond ranges, respectively. The goal is achieved by means of a nonlinear interferometric-type scheme based on the inverse process of PDC, i.e sum frequency generation (SFG). SFG has been used in classical and quantum optics as an ultrafast temporal correlator, e.g. to probe the temporal correlation of twin photons or twin beams [23–26]. In this work we investigate the correlation both

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in space and in time by monitoring, for different time delays and spatial separations of the signal and idler beams, the peak intensity of the coherent pump reconstructed via sum frequency mixing of the PDC radiation.

The experimental layout used here is shown in Fig.1. The possibility of resolving the spatio-temporal shape of the correlation relies on an achromatic imaging, which avoids dispersive optical elements that would drastically deteriorate the shape of the measured correlation [27].

Figure 1: (Color online) Experimental set-up used for the investigation of the non factorable spatio-temporal correlation of the twin beams.

The pump pulse at 527.5 nm, obtained from the second harmonic of a 1 ps, 1055 nm, 10 Hz repetition rate Nd:Glass laser (Twinkle, Light Conversion Ltd.), is collimated down to about 0.8 mm (FWHM) at the entrance of a type I 4 mm Beta Barium Borate (BBO) crystal for PDC in the collinear configuration. Just after the crystal, two custom made high reflectivity dielectric mirrors (Layertec) with reflectivity $R > 99\%$ in the 750-1300 nm range and $R \approx 0.3\%$ at $\lambda = 527.5nm$, are used to reflect the PDC radiation. The achromatic imaging from the output plane of the PDC crystal is performed by means of two identical 90 degree off-axis parabolic gold mirrors (OAP) collecting a huge portion of the spatio-temporal spectrum of the PDC radiation ($\geq 600nm$), as illustrated in [26]. The frequency mixing of the PDC radiation is performed in a second BBO crystal identical to the first. This SFG crystal is placed in the focal plane of the second OAP mirror and is mounted on a micrometer translation stage permitting to finely adjust its position relatively to the imaging plane of the system. Both crystals are also mounted on a rotation stage in order to adjust their orientation with the aim of working at exactly the same PM conditions.

A detailed theoretical analysis of this scheme is presented in Ref.[27], which describes how the full spatio-temporal PDC correlation can be investigated by means of the SFG process, by controlling both a temporal delay $\Delta t$ and a transverse displacement $\Delta \vec{x}$ inserted between twin beams. In the experiment this is realized by means of the two adjacent gold mirrors $M_1$ and $M_2$ (see Fig.1) placed in the far-field plane of the PDC crystal. They act separately on the twin components of the light, because each photon (signal) has its twin (idler) on the opposite side of the far-field due to momentum conservation in the elementary PDC process. While mirror $M_2$ can be translated by an amount $\Delta s$ to produce a relative delay $\Delta t = \sqrt{2\Delta s/c}$ between twin beams, the mirror $M_1$ can be rotated by an angle $\Delta \phi$ in order to generate at the SFG crystal input face a transverse displacement of one beam with respect to the other by an amount $\Delta x \approx 2f_{OAP}\Delta \phi$. A 2mm wide gap between the two mirrors allows to eliminate the residual input pump. Since the correlation is strongly localized both in space and in time (in the micrometer and femtosecond range respectively), the temporal and spatial relative displacements of the twin beams must be scanned with micrometric precision. This is done thanks to high resolution translation and rotation stage piezo-controllers (Mercury, PI).

The radiation emitted by the SFG crystal is observed in the focal plane of a $f=20$ cm focal length lens, where the light intensity is monitored by a 16 bit scientific CCD camera (Roper Scientific) with 80 % detection efficiency at 527.5 nm. In this plane we observe a narrow light peak originating from coherent up-conversion processes, and reproducing the far-field profile of the original pump [28], lying over a widely spread incoherent speckle field constituting the background.

The intensity of the coherent peak, monitored as a function of $\Delta \vec{x}$ and $\Delta t$ is able to give a precise information about the spatio-temporal correlation of twin beams (see [27]), namely about the modulus of the biphoton correlation $\Psi(\Delta \vec{x}, \Delta t) := \langle A(\vec{x}, t)A(\vec{x}+\Delta \vec{x}, t+\Delta t) \rangle$, where $A$ denotes the PDC field operator at the crystal output face. Such biphoton correlation represents the probability amplitude that two photons of a pair exit the crystal separated by $\Delta \vec{x}$ and delayed by $\Delta t$. In conditions close to collinear PM and for a broad enough pump beam, $\Psi(\Delta \vec{x}, \Delta t)$ assumes a X-shape in any plane containing the temporal delay $\Delta t$ and one spatial coordinate $\Delta x$ [20], expressing a strong relationship between the temporal delays and the spatial separations of twin photons:

$$\Delta t = \pm \sqrt{k'' \Delta \vec{x}}$$  \hspace{1cm} (1)$$

where $k'' = \left. \frac{d^2k}{dx^2} \right|_{\Omega=0}$ is the group velocity dispersion (GVD) coefficient. As shown in [21], the same geometry of the correlation persists in the high gain regime, where stimulated emission contributes to form twin beams.

The experimental results, obtained thanks to a careful control of the alignment criticity, are presented in Figs.2 and 3. In fig. 2, each plot, representing a temporal correlation profile, is associated with a given transverse spatial...
displacement of the twin beams at the SFG crystal input face (i.e. at the imaging plane of the output face of the PDC crystal). Here we present four examples of the temporal correlation profiles obtained by monitoring the SFG peak intensity as a function of the delay $\Delta t$ between the twin beams. Each data point corresponds to the coherent peak intensity averaged over 15 images, each of them recorded over 2s (20 laser shots). The single temporal correlation peak observed for $\Delta x = 0$ (Fig. 2a), splits for $\Delta x > 0$ into two peaks symmetrical with respect to $\Delta t = 0$, and whose temporal separation increases for increasing values of $\Delta x$ (Fig. 2b,c,d). The correlation time that characterizes the width of the two peaks is close to that found for the single peak at $\Delta x = 0$, about 6fs at FWHM. The experimental data agree satisfactorily with the theoretical calculations (solid red lines) obtained by means of the model presented in [27], and with parameters that simulate closely the experiment.

We notice that the rotation of mirror $M_1$ occurs around the "gap" axis between $M_1$ and $M_2$ (the y axis in Fig.1), so that at the SFG crystal input the twin components, separated at the $2f_{OAP}$ plane into a left and a right half-cone, are relatively displaced along the same left-right axis (the x-axis in Fig.1). For geometrical reasons, this choice implies that the probability of upconversion tends to vanish for $\Delta x < 0$, while it is enhanced for $\Delta x > 0$, so that the shape of the reconstructed correlation changes from an X to a V, as discussed in [27] and shown in Fig.3.

A full X-correlation could in principle be measured by rotating $M_1$ in the orthogonal direction, but with a visibility of the tails reduced [27] by a factor 1/4 [31]. In Fig 3, the positions of the correlation peaks obtained from an entire set of measurements, are reported in the $(\Delta x, \Delta t)$-plane, showing how they lie along the asymptotes of the X-structure (bold lines) $\Delta t = \pm \sqrt{k'' \Delta x}$ (Eq.(1)).

![Figure 2: Temporal correlation of twin beams reconstructed by monitoring the SFG peak intensity as a function of a temporal delay between the beams, for four different transverse spatial displacements $\Delta x$ at the SFG crystal input face (squares). The red solid lines are the theoretical predictions.](image)

This relation has its very origin in the PM mechanism, which imposes a balance between the GVD, responsible for separating temporally the twin photons during propagation, and the diffraction, which causes their transverse separation [29]. In order to elucidate this point, let us consider the mechanism of photon-pair generation. Energy-momentum conservation imposes that, in the limit of a plane-wave and monochromatic pump, twin photons are generated with opposite transverse wavevectors $\pm \vec{q}$, and with symmetric offsets $\pm \Omega$ with respect to the central frequency $\omega_0/2$. On the other hand, efficient down-conversion occurs only for those pairs satisfying longitudinal PM, $k_z(q, \Omega) + k_z(-q, -\Omega) - k_0 \approx 0$, where $k_z$ is the longitudinal component of their wave-vector and $k_0$ the pump wave-number. By making a Taylor expansion up to second order, the PM in collinear conditions can be casted as [21]

$$q^2 = k k'' \Omega^2,$$

expressing the well known feature of type I fluorescence cones, where photon pairs propagating at larger angles have larger frequency offsets. Let us now assume that a photon pair is generated at any point inside the crystal, at a distance $z$ from its output. During propagation the members of the pair can be delayed one with respect to the other because of GVD. Interpreting the two photons as two wave-packets centered around $\pm q$, with group velocities $v_g(\Omega) = dk(\Omega)/d\Omega$, the delay between twin photons at the crystal output is $\Delta t(z, q, \Omega) \approx z/v_g(\Omega) - z/v_g(-\Omega) \approx 2k'' \Omega$, where we
assumed small angles and frequency offsets (same limit of validity as (2)). Because of diffraction, twin photons separate in space: assuming they are e.g. emitted in the $(x,z)$ plane, and using again the analogy with classical wave packets, their transverse separation at the crystal exit is $\Delta x(z,\tilde{q},\Omega) = z \tan \theta(q,\Omega) - z \tan \theta(-q,\Omega) \approx 2 \tilde{q} q / k$ where $\theta$ is the angle between the propagation direction of the photon and the $z$-axis, and we assumed small angles, i.e. $\tan \theta(q,\Omega) \approx q / k$. By using now the PM relation (2), we easily obtain $\Delta y(z,\tilde{q},\Omega) = \pm \sqrt{k^3 / \Omega} \Delta x(z,\tilde{q},\Omega)$, valid irrespectively of the point where the photon pair was created and of the mode $\tilde{q},\Omega$, implying thus the proportionality between temporal delays and spatial separations expressed by (1). We remark that this relation has a much wider range of validity than that derived here, as clarified in [29].

In conclusion, we have experimentally demonstrated the non factorability of the spatio-temporal correlation of twin beams in a high gain PDC experiment. These results, having their origin in the PM mechanism which imposes a balance between GVD and diffraction, suggest that signal and idler photons do not necessarily exit from the PDC crystal at the same time and in the same place, but that their emission out of the medium is correlated in space-time along skewed lines. The result is once more a clear evidence of the importance of accounting for space-time along skewed lines. The result is once more a

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References